

NAME \_\_\_\_\_

i \_\_\_ ii \_\_\_ iii \_\_\_ iv \_\_\_ v \_\_\_ vi \_\_\_ vii \_\_\_ viii \_\_\_ ix \_\_\_ x \_\_\_ xi \_\_\_ xii \_\_\_ xiii \_\_\_ TOTAL \_\_\_\_\_

December 12,  
2003

Mathematics 206a  
Multivariable Calculus  
Final Examination

Mr. Haines

(3) I. Let  $\mathbf{a} = (1, 3, 7)$ ,  $\mathbf{b} = (0, 3, 6)$ , and  $C$  be the straight line segment connecting  $\mathbf{a}$  to  $\mathbf{b}$ . Give a parametrization for  $C$ .

(9) II. Let  $\mathbf{x}(t) = (t, t^2, t^3)$  from  $t = 0$  to  $t = 3$  be the parametrization of a curve in  $\mathbb{R}^3$ .

A. Give an equation of the tangent line to this curve at the point where  $t = 2$ .

B. Give the cosine of the angle between  $\mathbf{x}(t)$  and this tangent line at  $t = 2$ .

C. Set up but **do not evaluate** an integral whose value is the length of this curve.

(12) III. Give examples of:

A. Equations of two distinct parallel planes in  $\mathfrak{R}^3$ .

B. Parametric equations of two distinct parallel lines in  $\mathfrak{R}^4$ .

C. A non-constant vector field defined on  $\mathfrak{R}^3$  that is path independent.

D. A quadratic form.

(15) IV. Let  $M$  be the triangular surface in the plane  $\frac{x}{2} + \frac{y}{3} + z = 1$  that is cut off by the three coordinate planes. ( $M$  lies in the first octant, where  $x \geq 0, y \geq 0,$  and  $z \geq 0$  .)

A. Give a parametrization for the surface  $M$ .

B. Set up and evaluate an iterated integral that gives the area of  $M$ .

C. Set up and evaluate an iterated integral that gives the surface integral of the vector field  $\mathbf{F}(x, y, z) = (y, 0, 0)$  over the surface  $M$ .

(5) V. Explain why you cannot use The Fundamental Theorem of Line Integrals to evaluate the line integral of  $\mathbf{F}(x, y, z) = (x, x^2, x^3)$  over a path connecting  $(0, 0, 0)$  to  $(1, 1, 1)$ .

(7) VI. Evaluate the line integral  $\oint_C -xydx + x^2dy$ , where  $C$  is the boundary of the triangle cut from the first quadrant by the lines  $x = 2$ ,  $y = x$ , the  $x$ -axis, and the  $y$ -axis.

(7) VII. Calculate the value of the double integral  $\iint_R dA$ , where  $R$  is the region whose boundary is the circle parametrized by  $\mathbf{x}(t) = (2 + 3\cos t, 5 + 3\sin t)$  for  $0 \leq t \leq 2\pi$ .

(7) VIII. Set up but **do not evaluate** an iterated integral to compute the volume of the solid below the surface  $x^2 + y^2 + z = 3$  which lies above the region  $R$  which is the right triangle with vertices  $(0, 0)$ ,  $(0, 2)$ , and  $(1, 0)$

(7) IX. Suppose  $f(x, y, z) = x^2 y^3 + xy - z - 3y$ . Compute the line integral of  $\nabla f$ , the gradient of  $f$  along the straight line path connecting  $(0, 0, 0)$  to  $(1, 1, 1)$ .

(7) X. Find the equation (either parametric or rectangular) of the tangent plane to the surface whose equation is  $x^2 + y^2 + z = 3$  at the point  $(1, 1, 1)$ .

(7) XI. Give a parametrization of the curve in  $\mathfrak{R}^3$  that is the intersection of the surface  $x^2 + y^2 + z = 3$  with the plane  $z = 2$ .

(7) XII. Evaluate  $\iint_{\partial S} \mathbf{F} \cdot \mathbf{n} \, d\sigma$ , where  $\mathbf{F} = x\mathbf{i} + z^2\mathbf{j} + y^3\mathbf{k}$  and  $S$  is the solid box determined by the three coordinate planes, the plane  $x = 2$ , the plane  $y = 3$ , and the plane  $z = 4$ .

(7) XIII. State Stokes's Theorem.