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I _____ II _____ III _____ IV _____ V _____ VI _____ VII _____ TOTAL _____

October 24
2003

Mathematics 206a
Multivariable Calculus
Examination #2

Mr. Haines

(11) I. Find the maximum and minimum values of the function

$$f(x, y) = 1 + x - y \text{ on the set } [0, 1] \times [0, 1].$$

(21) II. Suppose $f(x, y, z) = x^2 y^3 + xy - z - 3y$ and $\mathbf{a} = (1, 1, 1)$.

A. $\nabla f(x, y, z) =$

B. $\nabla f(\mathbf{a}) =$

C. The directional derivative of f at \mathbf{a} in the direction parallel to $\mathbf{x} = (1, 2, 3)$ is

(28) III. Suppose $f : \mathfrak{R}^3 \rightarrow \mathfrak{R}^3$ such that

$$f(x, y, z) = (y, x^2, z) \text{ for all } (x, y, z) \in \mathfrak{R}^3 \text{ and that } \mathbf{a} = (1, 2, 3).$$

A. The Jacobian matrix of \mathbf{f} at \mathbf{a} ,

$$J\mathbf{f}(\mathbf{a}) =$$

B. The (total) derivative of \mathbf{f} at \mathbf{a} ,

$$(\mathbf{Df}(\mathbf{a}))(x, y, z) =$$

C. $\text{curl } \mathbf{f}(\mathbf{a}) =$

D. $\text{div } \mathbf{f}(\mathbf{a}) =$

(10) IV. Find the equation of the tangent plane at the point $(0, -1, 1)$ to the surface whose equation is

$$x^2 + 8y + 6z^2 = -2$$

(10) V. Calculate the Second Taylor polynomial at $\mathbf{a} = (1, 2)$ of the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ with formula

$$f(x, y) = x^3 + y^3$$

(10) VI. The point $(0, 2)$ is a critical point of $f(x, y) = 2x^2 + x^2y + y^2 - 4y$. Use the Second Derivative Test to determine whether $(0, 2)$ is a local minimum, a local maximum, or neither.

(10) VII. Suppose $f(x, y, z) = x + y^2 - z^3$ and $g(s, t) = (st, s - t, s + t)$. Use the chain rule to find the derivative of $f \circ g$ at the point $(1, 1)$.