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Mathematics 206a
Multivariable Calculus
Examination #1

Mr. Haines

- (10) I. P and Q are points in \mathcal{R}^3 , P has coordinates (1, 1, 1), and Q has coordinates (4, 5, 6).
What is the equation of the plane consisting of all points equidistant from P and Q?

(20) II. Give examples of:

A. Equations of two distinct parallel planes in \mathfrak{R}^3 .

B. An equation for any cone in \mathfrak{R}^3 .

C. Two different unit vectors in \mathfrak{R}^2 that are not perpendicular.

D. A parametric equation for any circle in \mathfrak{R}^3 that has radius 4 .

(10) III. $\mathbf{A}(t) = (1+t)\mathbf{i} + (\ln t)\mathbf{j} + \mathbf{k}$ is a parametrization of a path in 3-space. Give the equation of the tangent line to this path at the point where $t = 1$.

(5) IV. If you connect these points in \mathfrak{R}^3 in order you will see a quadrilateral:

$(0, 0, 0), (1, 3, 2), (3, -1, 4), (4, 2, 6), (0, 0, 0).$

Use the dot product to determine whether the diagonals of this quadrilateral are perpendicular.

(5) V. Give a coordinate equation for the plane containing the point $(1, 2, 3)$ which is perpendicular to the cross product of the vectors $\mathbf{v}_1 = \mathbf{i}$ and $\mathbf{v}_2 = \mathbf{j}$.

(10) VI. Suppose $f(x, y) = \frac{xy}{x^2 - y^2}$.

A. Sketch a graph of the the domain of f .

B. Is the domain open, closed, or neither? Explain your answer.

(5) VII. Here are the four corners of a parallelogram in \mathfrak{R}^2 :

(1, 1), (2, 4), (3, 2), and (4, 5).

What is its area?

(5) VIII. Suppose $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{T} : \mathfrak{R}^2 \rightarrow \mathfrak{R}^2$ is a linear transformation with the formula $\mathbf{T}(\mathbf{x}) = \mathbf{A}\mathbf{x}$. Suppose $\mathbf{a} = (3, 4)$. What is $\mathbf{T}^{-1}(\mathbf{a})$?

(10) IX. Given the quadratic form $r(x, y, z) = x^2 + 24yz + y^2 + z^2$

A. Express $r(x, y, z)$ in the form $(x, y, z)\mathbf{S}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, where \mathbf{S} is a symmetric matrix

B. Is $r(x, y, z)$ positive definite, negative definite, or indefinite? Explain why.

(20) X. Suppose $f(x, y) = x\sqrt{y+1}$

A. $\frac{\partial f}{\partial x}(x, y) =$

B. $\frac{\partial f}{\partial y}(x, y) =$

C. $\frac{\partial^2 f}{\partial x \partial y}(x, y) =$

D. $\frac{\partial^2 f}{\partial y \partial x}(x, y) =$