Math 206 Test One

Name: ____________________

1. (5 marks, level of difficulty 5) Answer one of **a.** or **b.**

a. Prove that if $f : U \subset R^n \to R$ is continuous at $\vec{a}$ and $f(\vec{a}) \neq b$ for some number b, then there exists $r > 0$ such that $f(\vec{x}) \neq b$ for all $\vec{x} \in B_r(\vec{a})$.

b. Use analysis to prove that $\lim_{(x,y)\to(0,0)} \frac{y^2}{\sqrt{x^2+y^2}} = 0$

2. (5 marks, level of difficulty 1) Answer one of **a.** or **b.**

a. Identify $C = \{(x, y) : -5 < x < 5\} \subset R^2$ as open, closed, or neither and find the boundary and complement.

b. Find (approximately) the maximum and minimum values of the function $h(x, y) = \sqrt{x^2 + y^2}$ on the set $S = \{(x, y) : x^2 + y^2 \leq 4\}$.

3. (5 marks, level of difficulty 2) Answer one of **a.** or **b.**

a. Show that $f(x,y) = e^{x-y}$ is a solution to the partial differential equation $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$.

b. Compute $\frac{\partial}{\partial x}(xe^y)$ using the **Definition of the partial derivative.**

4. (5 marks, level of difficulty 3) Answer one of **a.** or **b.**

a. Calculate the Jacobian matrix for the function $f(x, y) = 8x - 7y + 2$ at the point $\vec{a} = (-4, -5)$. Then write a formula for the total derivative.

b. Show that if $\vec{f} : U \subset R^n \to R^n$ is a linear transformation, then $(D\vec{f}(\vec{a}))(\vec{x}) = \vec{f}(\vec{x})$.

5. (5 marks, level of difficulty 4) Answer one of **a.** or **b.**

a. Use the chain rule to find the derivative of $\vec{g} \circ \vec{f}$ at the point $\vec{a} = (3, 2)$ where $\vec{g}(x, y) = (x^2y^3, 3x - y^2)$ and $\vec{f}(x, y) = (-y, x)$.

b. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ assuming that the equation $x^3y^2z + xy - z^3 = 0$ implicitly defines z as a function of x and y.