

**MATH206A MULTIVARIABLE CALCULUS - PROF. P.
WONG**

EXAM I - SEPTEMBER 27, 2007

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	Maximum Score	Your Score
1.	16	
2.	17	
3.	17	
4.	17	
5.	16	
6.	17	
Total	100	

1. Let P be the parallelogram in \mathbb{R}^3 with vertices

$$A = (1, -1, 2), B = (2, 0, 1), C = (3, 2, -1), \text{ and } D = (2, 1, 0).$$

[Don't spend too much time drawing the picture!]

(8 pts) (i) Find the area of P .

(8 pts) (ii) Let $E = (2, -2, 5)$. Find the volume of the parallelepiped spanned by the vectors \vec{AB} , \vec{AD} and \vec{AE} .

2. Consider the following two planes $P_1 : 4x - y + z = 2$ and $P_2 : 2x - z = 3$ in \mathbb{R}^3 .

(5 pts) (i) Find a point on the line of intersection between the planes P_1 and P_2 .

(3 pts) (ii) Find a vector orthogonal to the plane P_1 .

(3 pts) (iii) Find a vector orthogonal to the plane P_2 .

(6 pts) (iv) Find a parametrization for the line of intersection between P_1 and P_2 .

3. The position of a particle in \mathbb{R}^3 is given by the parametrization

$$r(t) = e^t \mathbf{i} + \mathbf{j} + \sin t \mathbf{k}, \quad \text{for } t \geq 0.$$

(5 pts) (i) Find the velocity $v(t)$ of the particle at time t .

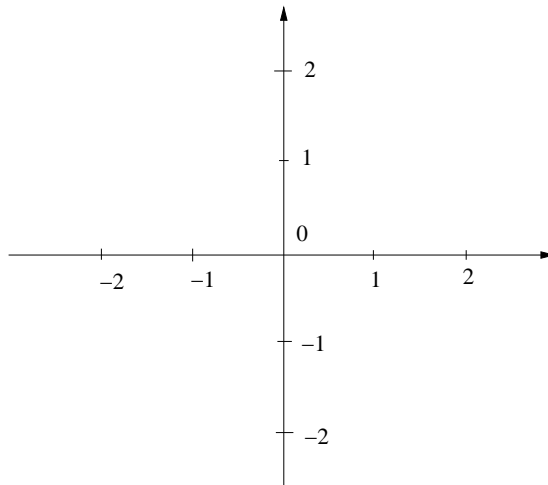
(6 pts) (ii) Find the projection of the initial velocity in the direction of $\mathbf{a} = \mathbf{i} + 2\mathbf{j}$.

(6 pts) (iii) Find the angle between the position and the velocity of the particle at $t = \frac{\pi}{4}$.

4.(4 pts) (i) Consider the line given by the parametric equations $x = t + 1$, $y = 2 - 3t$, $z = 2t + 1$. Write this line in vector form $\mathbf{x} = t\mathbf{m} + \mathbf{x}_0$

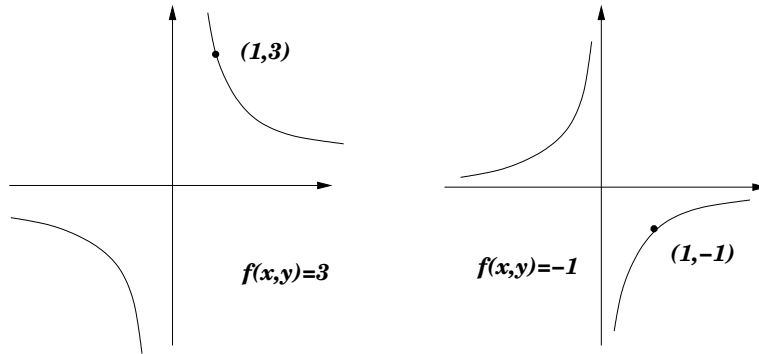
(8 pts) (ii) Give an equation for the plane that is perpendicular to the line given in (i) and that contains the point $(1, 1, 0)$. [Hint: use (i)]

(5 pts) (iii) Sketch the vector field $f(x, y) = (xy, 2)$ at the points $A = (1, -1)$, $B = (0, 1)$, $C = (-1, -1)$, $D = (1, 1)$.



5. Let $f(x, y) = xy$.

(5 pts) (i) Sketch the level curves of $f(x, y) = 3$ and $f(x, y) = -1$.

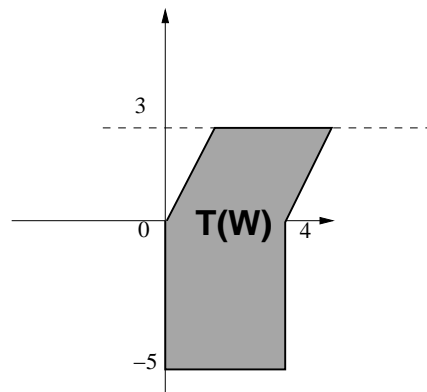


(5 pts) Describe or sketch the level set $h(x, y, z) = 4$ where $h(x, y, z) = x^2 + \frac{y^2}{9}$.

(6 pts) (iii) What are the cylindrical coordinates (r, θ, w) **AND** the spherical coordinates (ρ, θ, ϕ) of the point $(1, 1, \sqrt{2})$?

6. Let T be a linear transformation given by $T(x_1, x_2) = (3x_2 - x_1, 2x_1)$.
(7 pts)(i) Find the matrix A associated to T such that $T(\mathbf{x}) = A\mathbf{x}$.

- (10 pts)(ii) Suppose W is a figure in \mathbb{R}^2 so that $T(W)$ is the figure shown below.



- What is the area of the figure W ? [Hint: $T(W)$ is made up of a parallelogram and a rectangle; what is the relation between the area of W and the area of $T(W)$?]