Math 206, Section A

FINAL EXAM

04/13/07

1. (20 points) Find parametric equations of the line ℓ_2 through the point $Q\,(0,1,2)$ that is perpendicular to the line

$$\ell_1 : x = 1+t, y = 1-t, z = 2t$$

and intersects ℓ_1.

2. (20 points) Are the given lines intersecting, parallel, or skew? If they intersect, find the point of intersection. If they are parallel or skew find the distance between them.

$$\ell_1 : x = 2t, y = 1-t, z = -1+t \qquad \ell_2 : x = 2+t, y = 1-2t, z = 0$$

3. (20 points) Suppose we have a curve and a point on it, the **normal plane** to the curve at the point is the plane that contains the point and is perpendicular to the tangent line at the point. Find the point(s) on the curve $\mathbf{r}(t) = \left(t^3 - 4t\right)\mathbf{i} + t^5\mathbf{j} + (7t+2)\,\mathbf{k}$ where the normal plane is parallel to the plane $x - 5y - 7z = 17$. For each such point find an equation in linear form of the normal plane.

4. (20 points) Let $u = u\,(x,y,z)$ be a function of three variables. Suppose that $x = pq^2r$, $y = pq^3$, and $z = p^3$. Find $\dfrac{\partial^2 u}{\partial q \partial r}$.

5. (20 points) Find all up to the third degree terms of the Taylor series of $f\,(x,y) = x^2 \ln y$ at $(2,e)$, i.e., find the third Taylor polynomial of $f\,(x,y)$ at the given point.

6. (20 points) Find the global extrema of $f(x,y) = 2x^2 - 3y^2 - 2x$ on $\Omega = \left\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \le 1\right\}$.

7. (20 points) Minimize and maximize $x^4 + y^4 + z^4$ on the surface $x + y + z = 1$.

8. (20 points) Use polar coordinates to evaluate the iterated integral. $$\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{4-y^2}}^{-|y|} x^2 y^2 \, dx dy$$

9. (20 points) Find the area of the surface $z = x + y^2$ that lies over $D = \left\{(x,y) \in \mathbb{R}^2 : x \in [11,13]\,, y \in \left[0, \frac{\sqrt{6}}{2}\right]\right\}$.

10. (20 points) Use spherical coordinates to evaluate the triple integral where E is the solid bounded above by the surface $x^2 + y^2 - z^2 = 0$, below by the plane $z = 0$, and on the sides by the sphere $x^2 + y^2 + z^2 = 7$.

$$\iiint_E \frac{dxdydz}{x^2 + y^2 + z^2}$$

11. (20 points) Let S be a part of the graph of $z = e^{-x^2 - y^2}$ which is above the plane $z = 1/e$ with positive side on the top. Let $\mathbf{F}\,(x,y,z) = \langle xz, yz, e^{\arctan z}\rangle$. Use Stokes's Theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{n} d\sigma$.

12. (20 points) Let Ω be the boundary of the solid bounded by the surfaces $z^2 = 1$ and $x^2 + y^2 = 1$ with positive side outside. Let $\mathbf{F}\,(x,y,z) = \left\langle e^z \arctan y, x^3 y, x^3 \ln y\right\rangle$. Use Gauss's Theorem to evaluate $\iint_\Omega \mathbf{F} \cdot \mathbf{n} d\sigma$.