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XII___XIII___TOTAL____

April 9, 2008 | Mathematics 206a | Mr. Haines
Multivariable Calculus
Final Examination

(10) I. Give a parametrization for:

A. The line segment connecting the point (1, 3, 2, 4) and the point (2, 1, 5, 7) .

B. The plane containing the three points (1, 2, 3), (1, 4, 5), and (2, 7, 9).

(5) II. The point **a** = (1, 4, 3) lies on the sphere S of radius $\sqrt{26}$ centered at the origin. Give an equation of the tangent plane to S at **a** .

(5) III. Find $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4+y^4}$ if it exists. If it doesn't exist, explain why.

(15) IV. Give examples of

A. four vertices of a parallelogram that lies in a plane with equation $x+y+z=0$,

B. a function $f:\Re^3 \to \Re$ with level surface having equation. $x^4 - y^2 - z^2 - 4 = 0$,

C. a non-constant vector field on $\Re^3$ with divergence 0.

(10) V. Suppose $\boldsymbol{f}\,(x, y) = (x^2 - y^2,\ 2xy)$ and $\mathbf{a} = (1, 0)$.

A. Write a formula for D$\boldsymbol{f}(\boldsymbol{a})$, the derivative of $\boldsymbol{f}$ at $\boldsymbol{a}$.

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B. Describe in the words the action of D$\boldsymbol{f}(\boldsymbol{a})$ on vectors in $\Re^2$.

(5) VI. Calculate the value of $\int_C \vec{F} \bullet d\vec{x}$ if **F**(x, y, z) = -y**i** + x**j** + 5**k** and C is the curve parametrized by $\mathbf{c}(t) = (\cos t, \sin t, t)$ with $0 \le t \le 4\pi$.

(5) VII. If $f : \Re^2 \to \Re$ has rule $f(x, y) = x^2 + 2xy + 3y^2$, calculate the directional derivative of f at (2, 1) in the direction parallel to the vector **i** + **j**.

(5) VIII. Compute $\oint_C \mathbf{F} \bullet d\mathbf{x}$ where C is the boundary of the region in the first quadrant bounded by the curves $y = 0$, $x = 1$, and $y = x^2$

A. if $\mathbf{F}(x, y) = (y^2, y + x)$.

B. if $\mathbf{F}(x, y) = (2x + 3y, 3x + 2y)$.

(10) IX. Given the vector field **F** (x, y, z) = (*y*, *x*, 1)

A) Prove that **F** is path independent in $\Re^3$.

B) If C is a path in $\Re^3$ parametrized by **c** (t) = $(\cos(\pi t), \sin(\pi t), t)$ with $0 \le t \le 1$, use the Fundamental Theorem of Line Integrals to calculate $\int_C \mathbf{F} \bullet d\mathbf{x}$.

(5) X. If S is the solid bounded by $z = x^2 + y^2$ and $z = 4 - x^2 - y^2$, set up, but do not evaluate, the iterated integral that results from changing the triple integral $\iiint_S \left(1 + x\sqrt{x^2 + y^2}\right) dxdydz$ to cylindrical coordinates.
[Conversion formulas are: $x = r\cos\theta; y = r\sin\theta; z = w$.]

(10) XI. If $\mathbf{F}(x, y, z) = 3z\mathbf{i} + 5x\mathbf{j} - 2y\mathbf{k}$ and R is the part of the surface with equation $z = x^2 + y^2$ that lies below the plane $z = 4$,

A. give a parametrization of the boundary of R ,

B. use Stokes's Theorem to evaluate $\iint_R curlF \bullet n d\sigma$.

(10) XII. Find the area of the portion of the plane $z = x + 3y$ that lies inside the circular cylinder with equation $x^2 + y^2 = 4$.

(5) XIII. If $f : \Re^2 \to \Re$ with rule $f(x, y) = x^2 + xy + y^2$ calculate the Hessian of f at (1, 3).