

NAME _____

I ___ II ___ III ___ IV ___ V ___ VI ___ VII ___ VIII ___ IX ___ TOTAL _____

March 10
2004

Mathematics 206a
Multivariable Calculus
Examination #2

Mr. Haines

(5) I. Give an example of vector field on \mathfrak{R}^3 .

(10) II. Find the equation of the tangent plane at the point $(0, 1, 1)$ to the surface with equation:

$$x^3 - 5y + 6yz^2 = 1 .$$

(15) III. Suppose $f(x, y, z) = xy^2 + x^2y - z - 5x$ and $\mathbf{a} = (1, 1, 1)$.

A. $\nabla f(x, y, z) =$

B. $\nabla f(\mathbf{a}) =$

C. The directional derivative of f at \mathbf{a} in the direction parallel to the line $\mathbf{x}(t) = (t + 1, 3t + 2, 2t + 3)$ is

(10) IV. Calculate the second-degree Taylor polynomial centered at $\mathbf{a} = (1, 2)$ of the function

$f : \mathfrak{R}^2 \rightarrow \mathfrak{R}$ with formula

$$f(x, y) = xy + x^3 \quad .$$

(10)V. If $f(x, y) = x^3 + y^2$, give the Hessian form for f at $(1, 0)$.

(10)VI. For the vector field $\mathbf{F} = (xyz, xy^2 + z, 7)$

A) $\text{div}(\mathbf{F}) =$

B) $\text{curl}(\mathbf{F}) =$

(10)VII. Find all critical points of $f(x, y) = x^2 + y^2 - 4x$. Use the Second Derivative Test to determine whether each critical point is a local minimum, a local maximum, or neither.

(10)VIII. Suppose you learn that the Jacobian of a function f at \mathbf{a} is the matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 4 \end{bmatrix}.$$

Give a formula for $(Df(\mathbf{a}))(\mathbf{x})$ using no more than the symbols s, t, u, v, w, x, y, z , commas, and parentheses. Note: You don't have to use all of the symbols!

(10) IX. Suppose $f(x, y, z) = (y, z, x)$ and $g(s, t) = (st, s - t, s + t)$ Use the chain rule to find the derivative of $f \circ g$ at the point $(3, 2)$.