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I____II____III____IV____V____VI____VII____VIII____ IX____X____TOTAL ___________

February 17 2005 | Mathematics 206a Multivariable Calculus Examination #2 | Mr. Haines

(12)I. Suppose $f(x, y) = x\sin(xy)$

A. $\frac{\partial f}{\partial x}(x, y) =$

B. $\frac{\partial f}{\partial y}(x, y) =$

C. $\frac{\partial^2 f}{\partial x \partial y}(x, y) =$

D. $\frac{\partial^2 f}{\partial y \partial x}(x, y) =$

(8)II. Suppose $f : \Re^3 \to \Re^3$ with rule $f(x, y, z) = (xyz, xy, x)$.

A. Calculate $Jf(2, 2, 2)$, the Jacobian matrix of f at $(2, 2, 2)$.

B. Find a point at which $\boldsymbol{Df}(2, 2, 2,)$, the total derivative of f, has the value $(0, 8, 2)$.

(12) III. For the function $f(x, y, z) = x + y$ at the point (1, 1, 0) compute:

A) The Hessian matrix.

B) The Hessian form.

C) The second-degree Taylor polynomial.

(12) IV. Suppose $F:\Re^3 \to \Re^3$ with rule $F(x,y,z) = (x^2, y^2, x^2 - y^2)$ and $G:\Re^3 \to \Re^3$ with rule $G(x,y,z) = (x^2 + y^2 + z^2, x + y + z, z)$.

A. Calculate the Jacobian matrix of F at the point (1, 2, 1).

B. Calculate the Jacobian matrix of the function G at the point F (1, 2, 1).

C. Calculate the Jacobian matrix of the function G o F at the point (1, 2, 1).

(10) V. Find the equation of the tangent plane at the point (0, 1, 1) to the surface with equation:

$$x^3 - 5y^2 + 6yz^3 = 1 \quad .$$

(10) VI. Suppose $f(x,y,z) = xy^2 + x^2y - z - 5x$ and $\mathbf{a} = (1, 1, 1)$.

Compute the directional derivative of f at **a** in the direction parallel to the line

x(t)= (t + 1, t + 2, t + 3).

(10)VII. Find all critical points of $f(x,y) = x^2 + y^2 - 4x - 2y + 5$. Use the Second Derivative Test to determine whether each critical point is a local minimum, a local maximum, or neither.

(8) VIII. Suppose $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$.

A. What is the domain of f?

B. $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist, and is so nasty that given any number you pick from –1 to 1 you can find a line along which to approach (0, 0) and get your number. Find a value for k so that if you approach (0, 0) along the line $y = kx$ the limit of $f(x,y)$ will be 4/5 .

(8) IX Give an example [a sketch is sufficient] of:

A. An open set in $\Re^2$ that is not bounded.

B. A closed set in $\Re^2$ that is bounded.

(10) X. Suppose $f(x,y) = \sqrt{x^2 - y^2}$

A. What is the domain of f?

B. What is $\vec{\nabla} f(x,y)$?

C. Find all the points where the gradient of f is zero.

D. Find all the points where the gradient is undefined. [This is an infinite set.]

E. Find the minimum value of f and the values of x and y where it occurs.