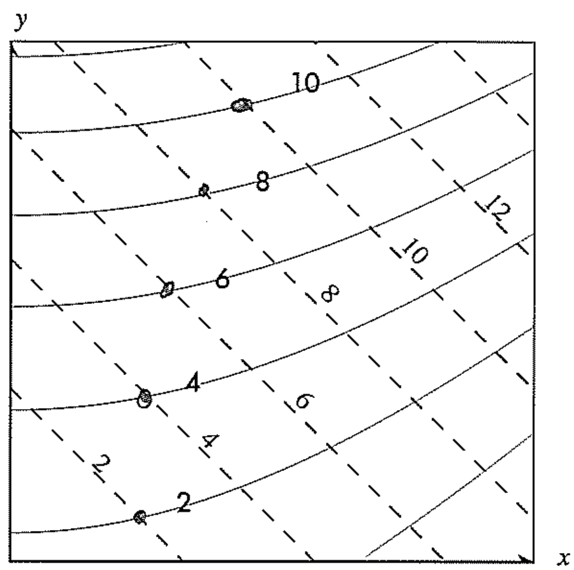
Name: Solutions

Math 206: Fall 2014
Exam 1

Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers. Correct answers without proper justification or those that use unapproved short-cut methods will not receive full credit. If you use a calculator to help find an answer, you must write down enough information on what you have done to make your method understandable.

Good Luck!

1. (4 points) The following figure shows contour diagrams of $f(x,y)$ (dashed lines) and $g(x,y)$ (solid lines). Plot 3 points where $f(x,y) = g(x,y)$.



2. (8 points) Find the x-slope and the y-slope of the plane containing the points $(-1,-2,0)$, $(-1,3,2)$, $(5,-2,4)$, and $(5,3,6)$.

y-slope: $(-1,-2,0) \to (-1,3,2)$ $\quad \frac{\Delta z}{\Delta y} = \frac{2-0}{3-{-2}} = \frac{2}{5}$

x-slope: $(-1,-2,0) \to (5,-2,4)$ $\quad \frac{\Delta z}{\Delta x} = \frac{4-0}{5-{-1}} = \frac{4}{6}$

3. ($\mathcal{8}$ points) For what value(s) of a is the vector $\vec{v} = -50\hat{\imath} + a\hat{\jmath} + 5\hat{k}$ perpendicular to the plane $z = 10x - 2y + 7$?

$\hookrightarrow 10x - 2y - z = -7 \implies \vec{n} = 10\hat{\imath} - 2\hat{\jmath} - \hat{k}$ is perp. to plane

$\vec{v} = \lambda\vec{n}$ makes $\vec{v}$ parallel to $\vec{n}$ hence makes $\vec{v}$ perpendicular to plane

$-50\hat{\imath} + a\hat{\jmath} + 5\hat{k} = 10\lambda\hat{\imath} - 2\lambda\hat{\jmath} - \lambda\hat{k}$

$-50 = 10\lambda \implies \lambda = -5$

$a = -2\lambda \implies a = (-2)(-5) = \boxed{10}$

4. ($\mathcal{8}$ points) Consider two vectors $\vec{v} = 5\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{w} = a\hat{\imath} + a\hat{\jmath} - \hat{k}$. For what value(s) of a are $\vec{v}$ and $\vec{w}$ perpendicular?

$\vec{v}$ and $\vec{w}$ are perp when $\vec{v} \cdot \vec{w} = 0$

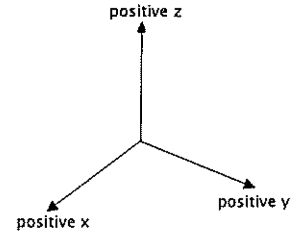
$\vec{v} \cdot \vec{w} = 5a - a - 3 = 4a - 3 = 0$

$\implies \boxed{a = 3/4}$

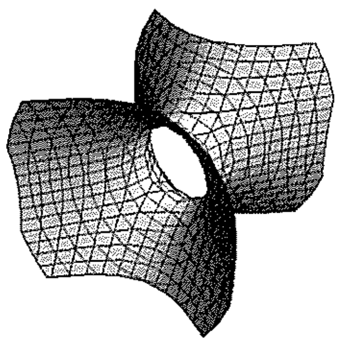
5. (20 points)

(a) Match each equation (in the table) with the graph of a surface. Enter O, if there is no match. The standard orientation of the axes applies in each graph.

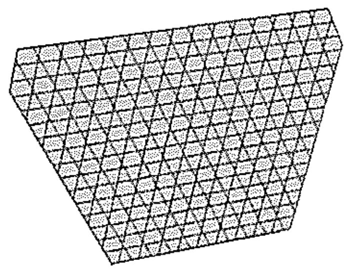
	O, I, II, III
$x + y^2 + z^2 = 1$	O
$x + y + z = 1$	II
$x + y^2 - z^2 = 1$	III
$-x^2 + y^2 + z^2 = 1$	I



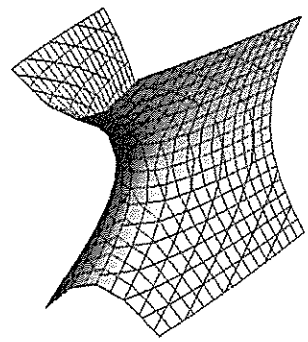
I



II



III



(b) For each equation, BRIEFLY justify your choice.

$x+y^2+z^2=1$: $x=0 \Rightarrow y^2+z^2=1 \Rightarrow$ circle
$z=0 \Rightarrow x+y^2=1 \Rightarrow$ parabola $\Big\} \Rightarrow$ none

b/c none of the graphs display circular cross-sections and parabolic level sets.

$x+y+z=1$ is linear which is a plane so II.

$x+y^2-z^2=1$: $x=0 \Rightarrow y^2-z^2=1 \Rightarrow$ hyperbola
$z=0 \Rightarrow x+y^2=1 \Rightarrow$ parabola $\Big\} \Rightarrow$ III
$y=0 \Rightarrow x-z^2=1 \Rightarrow$ parabola

we recognize a surface w/ parabolic slices in 2 directions and hyperbolic in the 3rd as a saddle, III is clearly a saddle

$-x^2+y^2+z^2=1$: $x=0 \Rightarrow y^2+z^2=1 \Rightarrow$ circle
$z=0 \Rightarrow -x^2+y^2=1 \Rightarrow$ hypebola $\Big\} \Rightarrow$ I

I has circular cross sections and hyperbolic level sets.

6. (16 points) Determine whether each of the following statements is true or false. BRIEFLY justify your answer.

(a) $(\hat{\imath} \times \hat{\jmath})$ and $(\hat{\imath} \times (\hat{\imath} \times (\hat{\imath} \times \hat{\jmath})))$ are parallel.

use the right hand rule

$\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\quad \hat{\imath} \times (\hat{\imath} \times (\hat{\imath} \times \hat{\jmath})) = \hat{\imath} \times (\hat{\imath} \times \hat{k}) = \hat{\imath} \times -\hat{\jmath}$
$= -\hat{k}$

since $-\hat{k}$ is a scalar multiple of $\hat{k}$, they are parallel. $\boxed{\text{TRUE}}$

(b) If $f(x, y)$ approaches 1 as (x, y) approaches $(0, 0)$ along the x-axis, then $\lim_{(x,y)\to(0,0)} f(x, y) = 1$.

$\boxed{\text{False}}$ It is possible to have a fcn $f(x,y)$ s.t. $f(x,y) \to 1$ as $(x,y) \to (0,0)$ along the x-axis and s.t. $f(x,y) \to 2$ as $(x,y) \to (0,0)$ along the y-axis. In this case $\lim_{(x,y)\to(0,0)} f(x,y)$ DNE.

(c) The two level surfaces $f(x, y, z) = 3$ and $f(x, y, z) = 5$ of the function $f(x, y, z) = 2x^2 + y^3 + z^4$ do not intersect at any point in space.

$\boxed{\text{TRUE}}$ If the level surfaces did intersect, say at (a,b,c), then we'd have $f(a,b,c) = 3$ <u>AND</u> $f(a,b,c) = 5$. But this is impossible since $f(x,y,z)$ is a function (one output only!)

(d) I love math! True. I have a PhD in math.