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I_____II_____III_____IV_____V_____VI_____VII_____VIII_____ TOTAL _____

February 6 2004 | Mathematics 206a
Multivariable Calculus
Examination #1 | Mr. Haines

(10) I. If $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k}$, give the equation of a line through the point (2, 5, 7) that is parallel to $\mathbf{a}$.

(10) II. Give a coordinate equation for the plane containing the point (1, 2, 5) which is perpendicular to the line with parametric equations $x = 3 + t$, $y = 7 - t$, $z = -1 + 3t$.

(20) III. A plane P in $\Re^3$ has equation $x - y + z = 0$.

A. Give a unit vector that is perpendicular to P.

B. Give a point that is in P.

C. Give the components of two non-parallel vectors lying in P.

D. Give a parametrization of P.

(10) IV. Here are the four corners of a parallelogram in $\Re^3$:

(0, 0, 0), (1, 3, 2), (2, 1, 3), and (3, 4, 5).

Use the dot product to show that the diagonals of this parallelogram are perpendicular.

(10) V. Calculate the integral:

$$\int \left(t^2 \mathbf{i} + (\sin t)\mathbf{j} + \left(\sqrt{\frac{1}{t}} \right) \mathbf{k} \right) \mathbf{d}t$$

(10) VI. Give examples of:

A. Two different unit vectors in $\Re^3$ that are not perpendicular.

B. A parametric equation for any circle in $\Re^3$ that has radius 4 .

(20) VII. Suppose $\mathbf{A} = \begin{bmatrix} 6 & 4 \\ 2 & 3 \end{bmatrix}$ and $\mathbf{T} : \Re^2 \rightarrow \Re^2$ is a linear transformation with the formula $\mathbf{T}(\mathbf{x}) = \mathbf{Ax}$.

Suppose $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{x_0} = (1, 4)$ and
$P = \{ \mathbf{x} \in \Re^2 \mid \mathbf{x} = \mathbf{x_0} + s\mathbf{a} + t\mathbf{b} , 0 \leq s, t \leq 1\}$.

A) What is the area of P?

B) What is the area of $\mathbf{T}(P)$?

(10)VIII. Given the quadratic form $r(x, y, z) = x^2 - 2yz + y^2 - z^2$

A. Express *r(x, y, z)* in the form $(x, y, z)\mathbf{S}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, where **S** is a symmetric matrix

B. Is *r(x, y, z)* positive definite, negative definite, or indefinite? Explain why.