

NAME \_\_\_\_\_

I\_\_ II\_\_ III\_\_ IV\_\_ V\_\_ VI\_\_ VII\_\_ VIII\_\_ IX\_\_ X\_\_ XI\_\_ XII\_\_ TOTAL\_\_  
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February 5  
2010

Mathematics 206  
Multivariable Calculus  
Examination #1

Mr. Haines

(15) I. If  $\mathbf{a} = \mathbf{i} + \mathbf{j}$  and  $\mathbf{b} = \mathbf{i} - 2\mathbf{j}$ , compute these:

A.  $\mathbf{a} \cdot \mathbf{b} =$

B.  $\|\mathbf{b}\| =$

C.  $\text{comp}_{\mathbf{b}}\mathbf{a} =$

D.  $\text{proj}_{\mathbf{b}}\mathbf{a} =$

(5) II. Give an equation of the plane containing the points  $(1, 2, 3)$  and  $(3, 6, 7)$  and with normal  $2\mathbf{i} - \mathbf{j}$ .

(10) III. Give examples of the following sets in  $\mathbb{R}^2$

A. A set that is open and bounded.

B. A set that is open and not bounded.

(10) IV.  $\mathbf{A}(t) = \left(1+t, t^2, \frac{1}{t}\right)$  with  $t \geq 1$  is a path in  $\mathfrak{R}^3$ .

A. Calculate  $\mathbf{A}'(t)$ , the derivative of  $\mathbf{A}(t)$  .

B. Give an equation of the tangent line to this path at the point where  $t = 1$ .

(5) V. Identify in words the surface whose equation is  $x^2 - y^2 - z^2 - 1 = 0$

(5) VI. Suppose  $\mathbf{a}$  is a vector with tail at the point  $(1,2,3)$  and head at the point  $(3,5,5)$ . Give a unit vector that is perpendicular to  $\mathbf{a}$ .

(5) VII. Compute the area of the parallelogram in  $\mathbb{R}^2$  with vertices  $(1,1)$ ,  $(5,7)$ ,  $(4,5)$ , and  $(2,3)$ .

(5) VIII. The plane P has coordinate equation  $2x + 3y + z = 5$ .

Give an equation for any line lying in P:

(10) IX. Give examples of:

A. Two unit vectors in  $\mathfrak{R}^3$  that are perpendicular.

B. Equations of two distinct parallel planes.

(15) X. If  $f(x, y) = x \sin y$

A.  $\frac{\partial f}{\partial x}(x, y) =$

B.  $\frac{\partial f}{\partial y}(x, y) =$

C.  $\frac{\partial^2 f}{\partial y \partial x}(x, y) =$

D.  $\frac{\partial^2 f}{\partial x \partial y}(x, y) =$

(10) XI. For the quadratic form

$$p(x, y, z) = -x^2 - 2y^2 - 5z^2 - 2xz ,$$

A. Give a symmetric matrix  $S$  that is the matrix of this quadratic form.

B. By taking determinants and using Sylvester's Theorem, determine if  $p$  is positive definite, negative definite, indefinite, or none of these.

(5) XII. A student says that any three points in  $\mathbb{R}^3$  determine a plane. She wants to find the equation of the plane that contains the points  $(1, 1, 3)$ ,  $(1, 0, 4)$ , and  $(1, -1, 5)$ . She knows she needs to find a normal to the plane, but has trouble computing it. Why?