Math 105a Review Problems for Final Exam

The exam will cover the following sections: 1.7, 2.1 - 2.7, 3.1 - 3.7, 3.9, 3.10, 4.1, 4.3, 4.5, 4.7, 5.1 - 5.4, 6.1 - 6.4. I may ask you to state one of the following:

Definitions

- limit definition of derivative (p. 79)
- continuity (p. 95)
- local linearization (p. 150)
- dominant function as \( x \to \infty \) (p. 157)
- average value of a function (p. 239)

Theorems

- Intermediate Value Theorem (p. 47)
- Extreme Value Theorem (p. 207)
- Mean Value Theorem (p. 208)
- Fundamental Theorem of Calculus Part I (Theorem 5.1 p. 244)
- Fundamental Theorem of Calculus Part II (Theorem 6.2 p. 279)

You are expected to know how to apply all other theorems and definitions. You will be expected to know all rules and properties for differentiation and antidifferentiation. The problems below are by no means a comprehensive list of the types you may encounter on the test. In fact, the problems below cover only the new material since the last test. Be sure to review all homework problems, quizzes, reviews, and tests in preparation for the final exam on Tuesday, April 13 at 8am.

1. Consider the graph of the derivative \( f' \) shown below. Sketch curves for two antiderivatives \( f \) such that, in the first case, \( f(0) = -1 \) and in the second case, \( f(0) = 2 \). Identify all critical points, local extrema, and inflection points.

2. The velocity of a slug, in feet per hour, crawling across the pavement is given by \( f(x) = \ln(x + 1) \).

   (a) Write an integral that represents the total distance traveled by the slug in the first 5 hours since it started moving.

   (b) Suppose you want to use upper and lower sums to estimate your integral in part (a). What should \( \Delta x \) to be in order to guarantee that the difference in upper and lower estimates is less than 1?

   (c) Using the value of \( \Delta x \) determined in part (b), find the upper and lower estimates for the integral representing total distance traveled. Is the difference in your estimates less than 1?

   (d) Find an estimate for the average velocity of the slug over the first 5 hours since it started moving.
3. Consider the region bounded above by the graph of \( f(x) = e^{-x^2} \), bounded below by the graph of \( g(x) = e^x - 1 \), and bounded on the left by the \( y \)-axis.

(a) Using your calculator, sketch and label the curves and shade the described region.

(b) Use your calculator to find an approximation for the coordinates of the intersection point of \( f(x) = e^{-x^2} \) and \( g(x) = e^x - 1 \).

(c) Use geometry to explain why the area of the shaded region is more than 0.3 and less than 0.7.

(d) Express the area of the shaded region as an integral. Approximate the value of the integral using left and right sums with \( n = 7 \).

4. Use the Fundamental Theorem of Calculus to find the area between the \( x \)-axis and the curve \( f(x) = (3x - 1)^2 \) between \( x = 0 \) and \( x = 2 \).

5. The graph of \( f(t) \) is given below.

![Graph of f(t)](image)

(a) Consider the area function \( F(x) = \int_0^x f(t) \, dt \). Complete the table of values for the function \( F(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Using the values from the table, sketch the graph of \( F(x) \).

(c) Find \( F'(4) \).

6. Find \( \frac{d}{dx} \left( \int_x^{0.57} \frac{1}{\ln(t^2)} \, dt \right) \).

7. Write an expression for the function, \( f(x) \), with the given properties: \( f'(x) = \sec(x^2) \) and \( f(0) = 2 \). Hint: Use the Fundamental Theorem of Calculus Part II.

8. Find the solution to the initial value problem: \( \frac{dy}{dt} = \cos(\pi t) \) with \( y\left(\frac{1}{2}\right) = 0 \).

9. Determine if \( y = x(1 - \ln x) \) is a solution to the differential equation \( \frac{dy}{dx} = -\ln x \) with initial value \( y(1) = 1 \).

10. On a distant planet, acceleration due to gravity is \(-10 \text{ ft/sec}^2\). Suppose a ball is thrown upward at time \( t = 0 \) at 30 feet/sec from a platform 35 feet high.

(a) When does the ball reach its highest point?

(b) Find a function representing the height of the ball at any time \( t \).

(c) What is the maximum height that the ball reaches?

(d) What is the total distance that the ball travels before hitting the ground?