NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

*Advice*: DON’T spend too much time on a single problem.

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<th>Problems</th>
<th>Maximum Score</th>
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1. Find the derivative $f'(x)$ for each of the following

(6 pts.) (a) $f(x) = (x + \sqrt{x}) \sin x$

(6 pts.) (b) $f(x) = \ln(x + 3x^3)$

(6 pts.) (c) $f(x) = \sqrt{x} + \tan x$
2. (20 pts.) A package can be sent by parcel post only if the sum of its length and girth (the perimeter of the base) is not more than 108 inches. Find the dimension of the box of maximum volume that can be sent, if the base of the box is a square.
(15 pts.) Consider a sports car which accelerates from 0 ft/sec to 88 ft/sec in 5 seconds (88 ft/sec = 60 mph). The car’s velocity is given in the table below.

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tr>
<td>v(t)</td>
<td>0</td>
<td>30</td>
<td>52</td>
<td>68</td>
<td>80</td>
<td>88</td>
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(8 pts.) (a) Using Riemann sums, find upper and lower bounds for the distance the car travels in 5 seconds.

(5 pts.) (b) Use your results in (a) to estimate \( \int_0^5 v(t) \, dt \).

(4 pts.) (c) Estimate the average velocity of the car over the five second time interval.
4. The graphs of $H(x)$ and $G(x)$ are shown in the figure. Let $F(x) = H(G(x))$ be the composite function [e.g. $F(0) = H(G(0)) = 2$]. Find

(a) $F(1)$

(b) $F'(1)$

(c) $F(3)$

(d) $F'(3)$. 
5. A particle is moving with acceleration $a(t) = 3t + 8$, initial position $s(0) = 1$ and initial velocity $v(0) = -2$.

(a) Determine the velocity function $v(t)$ at time $t$.

(b) Determine the position function $s(t)$ at time $t$.

(c) How far did the particle travel between $t = 1$ and $t = 2$?
6. (6 pts.) (a) Use implicit differentiation to find an equation of the line tangent to the graph of \( y \) at the point \((1, -2)\) where \( y^4 + 3y - 4x^3 = 5x + 1 \).

(6 pts.) (b) Evaluate the following limit.

\[
\lim_{x \to 1} \frac{\sin(\ln(x^2))}{x - \cos(x - 1)}.
\]

(6 pts.) (c) Find the indefinite integral

\[
\int (3\sqrt{x} - \sin x) \, dx.
\]
7. The following figure shows the graph of the derivative $f'$ of $f$ over the interval $[0, 6.5]$.

(a) For what values of $x$ does $f$ have a local maximum or minimum? Explain.

(b) For what values of $x$ does $f$ have an inflection point? Explain.

(c) For what intervals is $f$ increasing and for what intervals is $f$ decreasing? Explain.