(18 points) I. Calculate:

A. If \( f(x) = x \ln(x^2 + 1) \), then \( f'(x) = \)

B. If \( f(t) = \frac{\cos t}{\sqrt{t}} \), then \( f'(t) = \)

C. \( \frac{d}{dx} \left( \frac{a}{cx + d} \right) = \)

D. \( \int_1^{x+1} \frac{dx}{x} = \)

E. \( \int_1^3 \frac{t^2 + 1}{t} \, dt = \)
F. \[ \lim_{{x \to 0}} \frac{\sin x}{{e^x - 1}} = \]

(10 points) II. Using this table of values for the function \( f \),

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-2</td>
<td>-5</td>
<td>-2</td>
<td>0</td>
</tr>
</tbody>
</table>

A. estimate \( \int_0^{1.5} f(x) \, dx \)

B. estimate \( f'(1.0) \)
III. Suppose \( F(x) = \int_{1/2}^{x} \ln t \, dt \) for \( x > 0 \).

A. Estimate \( F(1) \).

B. Compute \( F'(2) \).

C. For what values of \( x \) is \( F \) decreasing?

IV. Consider the differential equation \( \frac{dy}{dx} = -\ln x \) with initial condition \( y(1) = 1 \).

Show either that \( y = x(1 - \ln x) \) is a solution or that it is not a solution.
(22 points) V. Here is a graph of $f'$, the derivative of the function $f$.

Warning: These questions are about $f, f'$, and $f''$. The graph you see above is the graph of $f'$.

A. The values of $x$ (if any) at which $f'$ is not differentiable:

B. The values of $x$ (if any) at which $f'$ is not continuous:

C. The interval(s) where $f$ increases:

D. The interval(s) where $f$ is concave down:

E. The inflection points (if any) of $f$: 
F. \[ \int_{0}^{6} f'(t)dt = \]

G. The average value of \( f' \) over the interval \([0, 6]\) :

H. If \( f(0) = 5 \), then \( f(4) = \)

I. \( f''(3) = \)

J. The critical points (if any) for \( f \):

K. The local minima (if any) for \( f \).

(6 points) VI. A snowstorm lasts 4 hours. Suppose \( f(t) \) is the rate of snowfall (in inches per hour) \( t \) hours after the start of the storm.

A. Write a sentence (including units) describing the quantity measured by
\[
\int_{2}^{4} f(t)dt .
\]

B. Write a sentence (including units) describing the quantity measured by \( f'(3) \).
(10 points) VII. You are to design a gutter (a long trough) by bending up the edges of a long strip of metal 12 inches wide so that the cross-section of the gutter is a rectangle. (A picture is below.) The amount of metal to bend up must be chosen to make the cross-sectional area as large as possible, so that the gutter will carry as much water as possible. What are the cross-sectional dimensions of the gutter that achieves this?

(8 points) VIII. A population of cells is growing in a laboratory. The number of cells $N$ at time $t$ (measured in hours from the beginning of the experiment) changes at a rate given by

$$\frac{dN}{dt} = 2735.2e^{-t}$$

A. Give all solutions to this differential equation.

B. Three hours after the experiment begins there are 10,000 cells. Give a formula for $N(t)$. 
(9 points) IX. Given the equation $x^2 y + xy^2 = 6$,

A. what is the value of $\frac{dy}{dx}$ at the point where $x = 2$ and $y = 1$?

B. Give the local linearization of this curve at the point $(2, 1)$.

C. Use your answer to part B to approximate $y$ when $x = 1.9$. 