Mathematics 106  
Exam II  
November 15, 2002

<table>
<thead>
<tr>
<th>Problem</th>
<th>Possible</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

You must show all work to receive credit.  
Give exact answers (ln 5, e^2) unless requested otherwise.
1. Give a clear argument to show whether each series converges or diverges.
   a) \( \sum_{n=1}^{\infty} \frac{\sqrt{99n^3 + n^5}}{2n^3 + 7n} \)

   b) \( \sum_{n=1}^{\infty} \frac{n^3}{n!} \)

   c) \( \sum_{n=1}^{\infty} \frac{1}{n^2 + \frac{1}{5}} \)

2. a) Use a second degree Taylor polynomial to approximate \( \sqrt{100.1} \). Leave your answer as a simplified fraction.

   b) What is the maximum possible error in your approximation above? Leave your answer as a simplified fraction.
3.  
   a) Use series to evaluate \( \lim_{x \to 0} \frac{\sin(5x) - 5x}{13x^3} \).

   b) What is the exact value of the series \( 1 + 5 + \frac{25}{2!} + \frac{125}{3!} + \frac{625}{4!} + \ldots \)?

   c) Each year, a miser spends 10% less money than he did the year before. If he spends $25,000 the first year and lives forever, how much money will he spend altogether?

   d) If \( f(x) = \cos(5x^2) \), compute \( f^{(2001)}(0) \). (This is the 2001st derivative evaluated at 0.) You do not need to evaluate terms such as 100! or \( 2^{100} \).

   e) If \( f(x) = \cos(5x^2) \), compute \( f^{(2000)}(0) \). (This is the 2000th derivative evaluated at 0.) You do not need to evaluate terms such as 100! or \( 2^{100} \).
4. Circle the appropriate letter to indicate whether each statement is true always (A), sometimes (S), or never (N).

a) If the individual terms of a series approach 0, the series converges.  
A S N

b) If the individual terms of a series approach 0.2, the series converges.  
A S N

c) If the Ratio Test gives a ratio of 1, the series converges.  
A S N

d) If a geometric series has a ratio of 1 and a nonzero first term, the series converges.  
A S N

e) If a geometric series has a ratio of 0.2, the series converges.  
A S N

f) If a series fails the Alternating Series Test, the series diverges.  
A S N

g) If the terms of a series approach 0 and alternate in sign, the series converges.  
A S N

h) The series \( \sum_{n=1}^{\infty} \frac{1}{n^p} \) converges if \( p > 1 \).  
A S N

i) The series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^p} \) converges if \( p > 0 \).  
A S N

j) The series \( \sum_{n=1}^{\infty} \frac{b^n}{n!} \) converges for any constant \( b \).  
A S N