1. (20 points) Calculate the following limits. Please show all work.

(a) \[ \lim_{x \to 1} \frac{x^4 + 6x^3 - 22x + 15}{x^4 + x^3 - 3x^2 - x + 2} \]

(b) \[ \lim_{x \to \infty} \frac{x^2}{e^{x^2}} \]
2. (20 points) Find \( \frac{dy}{dx} \) if \( x^7 + y^7 = 14x^2y^4 \). What is the slope of this curve at \((7, 7)\)?
3. (20 points) Consider the parametric curve $x = t^2 - 1, y = t^3 - t$.

(i) For what values of $t$ does $x = 0$?
(ii) For what values of $t$ does the curve go through the origin?
(iii) Calculate $\frac{dx}{dt}$, $\frac{dy}{dt}$, and $\frac{dy}{dx}$.
(iv) What slope or slopes does the curve have at the origin?

Extra credit: Find an equation of the curve that has $x$ and $y$ in it, but not $t$. 
4. (20 points) Suppose $f(x)$ is a function whose first two derivatives are

$$f'(x) = x^6 - 30x^5 + 288x^4 - 864x^3 = x^3(x - 12)^2(x - 6)$$
$$f''(x) = 6x^5 - 150x^4 + 1152x^3 - 2592x^2 = 6x^2(x - 12)(x - 9)(x - 4)$$

For what values of $x$ is $f(x)$ increasing, and for what values is it decreasing? For what values of $x$ is $f(x)$ concave up, and for what values is it concave down? What are the local maxima and minima (peaks and valleys) of $f(x)$? What are its inflection points? Explain.
5. (20 points) Calculate the following derivatives. Simplify your answers if possible.

(a) \( a(x) = \ln \left( \frac{\sin x}{\cos x} \right) \)

(b) \( b(x) = \ln \left( \frac{\cos x}{\sin x} \right) \)

(c) \( c(x) = \ln \left( \ln \left( e^{e^x} \right) \right) \)