While a final answer is important, you earn points for all the work leading to that answer, not just the answer itself. Show all your steps clearly so you will be eligible for the most partial credit. You may use a calculator, but no notes, books, or other students. Good luck!

1.) (10 pts.) For some positive constant \( C \), a patient’s temperature change, \( T \), due to a dose, \( D \), of a drug is given by

\[
T(D) = \left( \frac{C}{2} - \frac{D}{3} \right) D^2.
\]

What dosage maximizes the temperature change?

\[
T(D) = \frac{C}{2} D^2 - \frac{1}{3} D^3
\]

\[
T'(D) = 2 \cdot \frac{C}{2} D - 3 \cdot \frac{1}{3} D^2
\]

\[
= CD - D^2
\]

Set \( T'(D) = 0 \):

\[
CD - D^2 = 0
\]

\[
D(C - D) = 0
\]

\[
D = 0, C
\]

Which gives a max.

\[
T''(D) = C - 2D
\]

\[
T''(0) = C > 0 \implies \text{min at } D = 0
\]

\[
T''(C) = -C < 0 \implies \text{max at } D = C
\]
2.) (10 pts.) Compute \( s'(y) \) if \( s(y) = \sqrt[3]{(\cos^2 y + 3 + \sin^2 y)} \). SIMPLIFY COMPLETELY, and remember you can simplify both before and after computing the derivative.

\[
\begin{align*}
s(y) &= \sqrt[3]{3 + \sin^2 y + \cos^2 y} = \sqrt[3]{3 + 1} = \sqrt[3]{4}, \text{ a constant.} \\
\end{align*}
\]

\( s'(y) = 0 \)

3.) (10 pts.) Compute \( f'(z) \) if \( f(z) = \sqrt{5z} + 5\sqrt{z} + \frac{5}{\sqrt{z}} - \sqrt{\frac{5}{z}} + \sqrt{5} \).

\[
\begin{align*}
f'(z) &= \frac{1}{2} (5z)^{-\frac{1}{2}} \cdot 5 + 5 \cdot 5z^{-\frac{1}{2}} - 5 \cdot \frac{1}{2} z^{-\frac{3}{2}} \\
&\quad - \frac{1}{2} (\frac{5}{z})^{-\frac{1}{2}} \cdot (-5z^{-2}) + 0 \\
\end{align*}
\]

No simplifying necessary.
4.) (10 pts.) A square-bottomed box with no top has a fixed volume, \( V \). (NOTE: \( V \) is a CONSTANT.) What dimensions minimize the surface area?

Volume: \( V = x^2 h \)

Surface Area: \( A = x^2 + 4xh \)

We want to minimize surface area \( A \).

Before computing \( A' \), we need to put \( A \) in terms of one variable.

Use \( V = x^2 h \), remembering that \( V \) is a constant.

\[
\begin{align*}
h &= \frac{V}{x^2} \\
A &= x^2 + 4x \left( \frac{V}{x^2} \right) \\
A &= x^2 + \frac{4V}{x} \\
A' &= 2x - \frac{4V}{x^2} \\
\text{Set } A' &= 0
\end{align*}
\]

Check: \( A'' = 2 + \frac{8V}{x^3} \)

\[
A'' \left( \frac{3\sqrt{2}V}{2} \right) = 2 + \frac{8V}{2V} > 0
\]

So, surface area is a min. 

Dimensions:

\[
\begin{align*}
x &= \frac{3\sqrt{2}V}{2} \\
h &= \frac{V}{\left( \frac{3\sqrt{2}V}{2} \right)^2}
\end{align*}
\]
5.) (10 pts.) For \( x > 0 \), find and simplify the derivative of \( f(x) = \arctan x + \arctan(1/x) \). What does your result tell you about \( f \)?

\[
\begin{align*}
  f'(x) &= \frac{1}{1 + x^2} + \frac{1}{1 + \left(\frac{1}{x}\right)^2} \cdot \left(-\frac{1}{x^2}\right) \\
  &= \frac{1}{1 + x^2} + \frac{-1}{x^2 + 1} \\
  &= 0 \\
  f'(x) = 0 \quad \text{means that } f(x) \text{ is a constant.}
\end{align*}
\]

6.) (10 pts.) Find \( \frac{dy}{dx} \) if \( x^2 + xy - y^2 = xy^2 \).

\[
\begin{align*}
  2x + (x \frac{dy}{dx} + y) - 3y^2 \frac{dy}{dx} &= x \cdot 2y \frac{dy}{dx} + y^2 \\
  \text{Get } \frac{dy}{dx} \text{ all on one side:} \\
  x \frac{dy}{dx} - 3y^2 \frac{dy}{dx} - 2xy \frac{dy}{dx} &= y^2 - 2x - y \\
  \text{Divide:} \\
  \frac{dy}{dx} &= \frac{y^2 - 2x - y}{x - 3y^2 - 2xy}
\end{align*}
\]
7.) (10 pts.) Find the tangent line approximation to \((1 + x)^{3/2}\) near \(x = 0\).

\[
f(x) = (1 + x)^{3/2}, \quad a = 0
\]

\[
f'(x) = \frac{3}{2} (1 + x)^{-\frac{1}{2}}
\]

\[
f(0) = 1, \quad f'(0) = \frac{3}{2}
\]

\[
f(x) \approx f(0) + f'(0) (x - 0)
\]

\[
f(x) \approx 1 + \frac{3}{2} x
\]

8.) (10 pts.) Compute the following limits:

a.) \(\lim_{x \to 0} \frac{e^x}{x}\)

\[
= \frac{e^2}{2}
\]

b.) \(\lim_{x \to \infty} \frac{e^x}{x}\)

\[
\lim_{x \to \infty} e^x = \infty, \quad \lim_{x \to \infty} x = \infty, \text{ so we can use L'Hopital's Rule}
\]

\[
\lim_{x \to \infty} \frac{e^x}{x} = \lim_{x \to \infty} \frac{e^x}{1} = \infty
\]
9.) (10 pts.) Sketch the graph of $f$ given that:

- $f'(x) = 0$ at $x = 2$, $f'(x) < 0$ for $x < 2$, $f'(x) > 0$ for $x > 2$,
- $f''(x) = 0$ at $x = 4$, $f''(x) > 0$ for $x < 4$, $f''(x) < 0$ for $x > 4$.

10.) (10 pts.) **True or False:** If $a < b$ and $f'(x)$ is positive on the closed interval $[a, b]$ then $f(a) < f(b)$. Justify your answer with a full explanation.

**True.**

**MVT:** IF

$f(x)$ is continuous on $[a, b]$  
$f(x)$ is differentiable on $(a, b)$

(there are both true: since $f'$ is positive, $f'$ exists and $f$ is differentiable, if $f$ is differentiable, then $f$ is continuous)

THEN there exists $c$ between $a$ and $b$ so that

$f'(c) = \frac{f(b) - f(a)}{b - a}$  
Since $f'$ is positive, $\frac{f(b) - f(a)}{b - a} > 0$.

for all $c$ between $a$ and $b$.

Since $b > a$, then $f(b) > f(a)$. 