NAME: 

Show ALL your work CAREFULLY.

A rectangular sheet of paper is to contain 72 square inches of printed matter with 2 inch margins at top and bottom and 1 inch margins on the sides. What dimensions for the sheet will use the least paper.

Let $H$ and $W$ denote the height and the width of the printed matter area respectively (shown in the figure). Then we know that $HW = 72$ and the area of the sheet of paper is given by $A = (H + 4)(W + 2)$ since the height of the paper is now $H$ plus the top and bottom margins and likewise the width of the paper is $W$ plus the side margins. To minimize $A$, we first rewrite $A$ in terms of $W$ (or of $H$).

$$A = \left( \frac{72}{W} + 4 \right)(W + 2)$$

$$= 72 + \frac{144}{W} + 4W + 8.$$ 

By taking the derivative, we have

$$A' = -144W^{-2} + 4.$$ 

By setting $A'$ to 0, we have

$$W^2 = \frac{144}{4} = 36.$$ 

Since $W$ cannot be negative, we have

$$W = 6$$ 

and hence, $H = 12$.

The desired dimensions are 16 inches (height) by 8 inches (width).

Date: November 10, 2004.