(10) I. Suppose \( A = \begin{bmatrix} 0 & 3 & -6 & 6 & 0 & -5 \\ 3 & -7 & 8 & -5 & 3 & 9 \\ 3 & -9 & 12 & -9 & 3 & 15 \end{bmatrix} \)

A. If \( \text{col } A \) is a subspace of \( \mathbb{R}^m \), what is the value of \( m \)?

B. If \( \text{nul } A \) is a subspace of \( \mathbb{R}^m \), what is the value of \( m \)?

(5) II. Give an example of a two-dimensional subspace of \( \mathbb{R}^4 \). Use correct mathematical notation to describe it.
III. If \( A = \begin{bmatrix}
1 & -1 & 0 & 3 & -2 \\
0 & 1 & 0 & -4 & 7 \\
0 & 0 & 1 & 0 & 6 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}, \)

A. Find a basis for \( \text{Col } A \).

B. Find a basis for \( \text{Nul } A \).

C. What is the dimension of \( \text{Col } A \)?

D. What is the dimension of \( \text{Nul } A \)?

E. What is the rank of \( A \)?
(10) IV. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ reflects points through the line $x_1 = x_2$,

A. What is the determinant of the standard matrix of the linear transformation $T$?

B. How many pivot positions does this matrix have?

(5) V. Because $AA^{-1} = I$, it follows that $(\det A)(\det A^{-1}) = \det I$. If $A = \begin{bmatrix} 7 & 1 & 1 \\ 0 & 2 & 5 \\ -7 & -1 & 1 \end{bmatrix}$, calculate $\det A^{-1}$. 
VI. Suppose that $B$ is obtained from $A$ by interchanging the first two rows of $A$, and that $\det(A) = \det(B)$. What is the value of $\det(A)$?

VII. Give an example of a matrix $A$ whose null space, $\text{Nul } A$, is a straight line in $\mathbb{R}^3$. 


(10) VIII. Compute the area of the parallelogram whose vertices are the points 
(4, 5), (1, 1), (2, 4), and (3, 2).

(10) IX. Suppose \( \mathbf{A} \mathbf{B} = \begin{bmatrix} -3 & 7 \\ 2 & 4 \end{bmatrix} \) and \( \mathbf{B} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \). Find \( \mathbf{A} \).
(15) X. Consider the production model \( x = Cx + d \) for an economy with two sectors, where

\[
C = \begin{bmatrix}
0.0 & 0.5 \\
0.6 & 0.3
\end{bmatrix}.
\]

A. Compute the matrix \( I - C \).

B. Compute the inverse of the matrix \( I - C \).

C. Use this inverse to determine the production level necessary to satisfy the final demand \( d = \begin{bmatrix} 20 \\ 10 \end{bmatrix} \).

(5) XI. TRUE OR FALSE? (Don't guess! The number of incorrect responses will be subtracted from the number of correct ones. Thus, random guessing earns you no points at all.)

______ 1. If \( A \) is a \( 2 \times 2 \) matrix with a zero determinant, then one column of \( A \) is a multiple of the other.

______ 2. It is possible to have two matrices \( A \) and \( B \) that are invertible, but their product is not invertible.

______ 3. If \( AB = AC \), then \( B = C \) for all matrices \( A \), \( B \), and \( C \).

______ 4. If \( H \) is a subspace of \( \mathbb{R}^m \) and \( \dim(H) = 4 \), then \( m \) must be greater than or equal to 4.

______ 5. If \( H \) is a subspace of \( \mathbb{R}^m \) and \( \dim H = 4 \), then \( H \) could have a basis with 2 elements.