Answer Key for Quiz 5 (section D)

1. If \( y = \sin^5 x \cos^4 x \), then the product rule says that

\[
\frac{dy}{dx} = \sin^5 x \cos^4 x \left( 4 \cos^3 x \frac{d}{dx} \cos x + 5 \sin^4 x \frac{d}{dx} \sin x \right).
\]

Then the chain rule says that

\[
\frac{dy}{dx} = \sin^5 x \cos^4 x \left( 4 \cos^3 x \sin x + 5 \sin^4 x \cos x \right)
\]

Some common factors could be taken out of this, but that would not produce a more aesthetically pleasing answer.

2. If \( a(x) = (\sin (e^x))^2 \) and \( b(x) = (\cos (e^x))^2 \), then \( a(x) + b(x) = 1 \), since \( \sin^2 \theta + \cos^2 \theta = 1 \) for any \( \theta \), including \( \theta = e^x \). Therefore the derivatives of \( a(x) \) and \( b(x) \) should be negatives of each other. We can confirm this by calculating both of them by the chain rule:

\[
a'(x) = 2 (\sin (e^x)) \frac{d}{dx} (\sin (e^x)) = 2 \sin (e^x) \cos (e^x) e^x \]

and similarly

\[
b'(x) = 2 (\cos (e^x)) \frac{d}{dx} (\cos (e^x)) = -2 \cos (e^x) \sin (e^x) e^x \]

so indeed we have \( b'(x) = -a'(x) \).