1. Let \( C = \begin{bmatrix} 3 & 7 & w & 2 \\ 9 & 5 & 1 & 3 \\ 0 & 2 & 4 & 6 \end{bmatrix} \) let \( D = \begin{bmatrix} 8 & 3 \\ 1 & 5 \\ 2 & 0 \\ 0 & 11 \end{bmatrix} \). Suppose the 1-1 entry of the product \( CD \) is 51.

Find each of the following values. If the value doesn’t exist write “DNE” in the box, and give the reason.

a. The number of entries in \( CD \).
   \( \text{Size is } (3 \times 4) \times (4 \times 2) \Rightarrow 3 \times 2 = 6 \)
   \( \text{DNE} \)

b. The 2-1 entry of \( CD \).
   \( 9 \cdot 8 + 5 \cdot 1 + 1 \cdot 2 + 3 \cdot 0 = 72 + 5 + 2 + 0 = 79 \)

d. The value of \( w \).
   \( 3 \cdot 8 + 7 \cdot 1 + w \cdot 2 + 20 = 51 \)
   \( 2w = 51 - 29 - 7 = 20 \)
   \( w = 10 \)

f. The 2-3 entry of \( (DC)^T \).
   \( \text{The product } DC \text{ is undefined. DNE} \)

h. The size of the matrix product \( CC^T \).
   \( (3 \times 4) \times (4 \times 3) = 3 \times 3 \)

i. The 2-1 entry of \( D^T \). \( 29 \)

j. The 1-2 entry of \( D^T \). \( 29 \)

2a. Find \( A^{-1} \) on your calculator using any method you like. Give your answer in fractions!

\[
A^{-1} = \begin{bmatrix} 375/4 & -205/6 & 99/6 \\ -21 & 23/2 & -11/3 \\ 11/2 & -2 & 1 \end{bmatrix}
\]

(\( \text{done by sifting } \) \( \text{T.I.} [A] \) \( \text{to } A \), \( \text{then finding } [A]^{-1} \))

2b. Solve \( Ax = b \). Give your answer in fractions!

\[
\hat{x} = A^{-1} b = \begin{bmatrix} \frac{4}{2} \\ \frac{5}{2} \end{bmatrix}
\]

(by sifting \( [B] \) to \( \begin{bmatrix} 22 & 8 \\ -6 & 3 \end{bmatrix} \) in T.I., then computing \( [A]^{-1} [B] \))

3. Let \( A \in M_{m \times n} \). Give four statements which are equivalent to the statement “\( A \) is invertible”. (From the invertible matrix theorem). Note that “\( A^{-1} \) exists” is not one of them.

Hints: span, linearly independent, RREF, solutions of \( Ax = 0 \), pivot, solutions of \( Ax = b \), associated linear transformation, …..

The hints reminded us of:

3a. The columns of \( A \) span \( \mathbb{R}^n \).
3b. The columns of \( A \) form a linearly independent set.
3c. \( \text{RREF}(A) = I_n \).
3d. The only solution of \( A \hat{x} = 0 \) is the trivial solution.

3e. \( A \) has \( n \) pivot positions.
3f. \( A \hat{x} = b \) has at least one solution for each \( b \in \mathbb{R}^n \).