1. If \( f(x) = \ln(e^x) \), then the chain rule says that

\[
f'(x) = \frac{1}{e^x} \cdot \frac{d}{dx} e^x = \frac{1}{e^x} e^x = 1.
\]

This is not surprising, since \( \ln x \) and \( e^x \) are inverse functions of each other, which means that \( \ln(e^x) = x \). Hence \( f(x) = x \), and therefore \( f'(x) = 1 \).

2. We have \( g(x) = \begin{cases} 
x^2 + 6x + 7 & \text{if } x < 1 \\
x^2 + 7x + 6 & \text{if } 1 < x < 3 \\
13x - 3 & \text{if } x \geq 3.
\end{cases} \)

(a) \( g(x) \) is not continuous at \( x = 1 \), because it is not defined at \( x = 1 \). This is so even though \( \lim_{x \to 1^-} g(x) = 1^2 + 6 \cdot 1 + 7 = 14 \) and \( \lim_{x \to 1^+} g(x) = 1^2 + 7 \cdot 1 + 6 = 14 \) also. Thus \( \lim_{x \to 1} g(x) \) exists and equals 14, but there is no \( g(1) \) for it to be equal to.

At \( x = 3 \) we have \( \lim_{x \to 3^-} g(x) = 3^2 + 7 \cdot 3 + 6 = 36 \) and \( \lim_{x \to 3^+} g(x) = 13 \cdot 3 - 3 = 36 \), so \( g(x) \) exists and equals 36, and also \( g(3) \) is defined; it equals \( 13 \cdot 3 - 3 = 36 \). Therefore \( g(x) \) is continuous at \( x = 3 \) since the limit equals the function value there.

(b) \( g'(x) = \begin{cases} 
2x + 6 & \text{if } x < 1 \\
2x + 7 & \text{if } 1 < x < 3 \\
13 & \text{if } x \geq 3.
\end{cases} \)

It is pretty obvious that the first two pieces of this are never going to agree. As \( x \to 1^- \), \( g'(x) \to 2 \cdot 1 + 6 = 8 \), and as \( x \to 1^+ \), \( g'(x) \to 2 \cdot 1 + 7 = 9 \). This tells us that \( g'(1) \) does not exist, but actually we knew that already—since there is no \( g(1) \), it is not possible that \( g'(1) \) could exist.

As \( x \to 3^- \), \( g'(x) \to 2 \cdot 3 + 7 = 13 \), and for any \( x > 3 \) (so in particular as \( x \to 3^+ \)) \( g'(x) = 13 \), so \( g'(3) \) exists and equals 13. We could therefore change \( 1 < x < 3 \) to \( 1 < x \leq 3 \) and \( x > 3 \) to \( x \geq 3 \) in the expression for \( g'(x) \), if we want.

(c) \( g''(x) = \begin{cases} 
2 & \text{if } x < 1 \\
2 & \text{if } 1 < x < 3 \\
0 & \text{if } x \geq 3.
\end{cases} \)

Since \( g(1) \) and \( g'(1) \) do not exist, \( g''(1) \) does not exist, even though it equals 2 on either side of \( x = 1 \). \( g''(3) \) does not exist either since \( 2 \neq 0 \).

4(a) If \( a(x) = x \cdot |x| \), then the product rule says that

\[
a'(x) = x \frac{d}{dx} |x| + |x| \frac{d}{dx} x = x \frac{|x|}{x} + |x| \cdot 1 = |x| + |x| = 2|x|.
\]

4(b) If \( b(x) = |e^x| \), then by the chain rule

\[
b'(x) = |e^x| \frac{d}{dx} e^x = |e^x| e^x = |e^x|.
\]

This is not too surprising since \( e^x \) is always positive; therefore \( |e^x| = e^x \) for all \( x \). So in fact \( b(x) = e^x \), and hence \( b'(x) = e^x \).
5(i) If \( c(x) = \left(x^8 - 6x^5 + 3 - 4x^{-3}\right)^6 \left(x^{12} + 7x^7 + 3x^{-8}\right)^9 \), then by the product rule and the chain rule we have

\[
c'(x) = \left(x^8 - 6x^5 + 3 - 4x^{-3}\right)^6 \frac{d}{dx} \left(x^{12} + 7x^7 + 3x^{-8}\right)^9 + \left(x^{12} + 7x^7 + 3x^{-8}\right)^9 \frac{d}{dx} \left(x^8 - 6x^5 + 3 - 4x^{-3}\right)^6
\]

\[
= \left(x^8 - 6x^5 + 3 - 4x^{-3}\right)^6 \left\{ 9 \left(x^{12} + 7x^7 + 3x^{-8}\right)^8 \frac{d}{dx} \left(x^{12} + 7x^7 + 3x^{-8}\right) \right\}
\]

\[
+ \left(x^{12} + 7x^7 + 3x^{-8}\right)^9 \left\{ 6 \left(x^8 - 6x^5 + 3 - 4x^{-3}\right)^5 \frac{d}{dx} \left(x^8 - 6x^5 + 3 - 4x^{-3}\right) \right\}
\]

\[
= 9 \left(x^8 - 6x^5 + 3 - 4x^{-3}\right)^6 \left(x^{12} + 7x^7 + 3x^{-8}\right)^8 \left(12x^{11} + 49x^6 - 24x^{-9}\right)
\]

\[
+ 6 \left(x^{12} + 7x^7 + 3x^{-8}\right)^9 \left(x^8 - 6x^5 + 3 - 4x^{-3}\right)^5 \left(8x^7 - 30x^4 + 12x^{-4}\right)
\]

(ii) If \( d(x) = e^{c(x)} \), where \( c(x) \) is the function in (i), then the chain rule says that \( d'(x) = e^{c(x)} c'(x) \). Since \( c(x) \) and \( c'(x) \) are given in (i), there is no need to write anything more.

Graphs for problem 3:

**Scores:** The median was 90, and the mean was 87.148148148148148148148148148148148148148148...