Mathematics 106  
Exam I  
October 10, 2002

<table>
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<th>Problem</th>
<th>Possible</th>
<th>Actual</th>
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<td>Total</td>
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You must show all work to receive credit.
No calculators are permitted.
Give exact answers (\(\ln 5, e^2\)) unless requested otherwise.

Some (possibly) useful formulae:

- area of circle = \(\pi r^2\)
- circumference of circle = \(2\pi r\)
- volume of sphere = \(\frac{4}{3}\pi r^3\)
- surface area of sphere = \(4\pi r^2\)
- volume of cylinder = \(\pi r^2h\)
1. Evaluate the following integrals.
   a) \[ \int_{x}^{\infty} \frac{3dx}{x(\ln x)^{2}} \]
   b) \[ \int x \sin(7x^2) dx \]
   c) \[ \int \cos(\sqrt{x}) dx \]

2. A spherical container of radius 8 feet is buried 7 feet below ground level and filled to a height of 5 feet with gasoline (42 pounds/cubic foot). Write (but do NOT evaluate) an integral equal to the
   a) total volume of the gasoline.
   b) work done in pumping all the gasoline to ground level.
3. Use an explicit inequality (< or >) to show each of the following converges or diverges.

a) \[ \int_{1}^{\infty} \frac{\sqrt{4x^4 + 5x^5}}{13x^3 + 2x^4} \, dx \]

b) \[ \int_{0}^{b} \frac{2x + 3x^2}{4x^3 + 7x^4} \, dx \]

4. Suppose that you invest money in a steady stream at a rate of $8000 per year. At 7% continuously compounded interest, when will you have a total of $400,000? You should solve for the time exactly, but you do not need to convert your answer to a decimal since you are not using a calculator.
5. Circle the appropriate letter to indicate whether each statement is true always (A), sometimes (S), or never (N).

a) \( \int_{1}^{\infty} \frac{1}{x^p} \, dx \) converges. A S N

b) If \( \lim_{x \to \infty} f(x) = 0 \) (that is, \( f(x) \to 0 \) as \( x \to \infty \)), then \( \int_{1}^{\infty} f(x) \, dx \) converges. A S N

c) If \( f \) is decreasing and concave down, then RHS(50) < \( \int_{a}^{b} f(x) \, dx \) < LHS(50). A S N

d) If \( f \) is decreasing and concave down, then RHS(50) < TRAP(50) < SIMP(50) < \( \int_{a}^{b} f(x) \, dx \) < MID(50) < LHS(50). A S N

e) If \( f \) is decreasing and concave down, then RHS(50) < TRAP(50) < SIMP(50) < \( \int_{a}^{b} f(x) \, dx \) < MID(50) < LHS(50). A S N

f) If \( 0 < f(x) < g(x) \) for all \( x \) and \( \int_{1}^{\infty} g(x) \, dx \) diverges, then \( \int_{1}^{\infty} f(x) \, dx \) diverges. A S N

g) If \( 0 < f(x) < g(x) \) for all \( x \) and \( \int_{1}^{\infty} g(x) \, dx \) converges, then \( \int_{1}^{\infty} f(x) \, dx \) converges. A S N

h) If \( 0 < f(x) < g(x) \) for all \( x \) and \( \int_{1}^{\infty} f(x) \, dx \) converges, then \( \int_{1}^{\infty} g(x) \, dx \) converges. A S N

i) If \( 0 < f(x) < g(x) \) for all \( x \) and \( \int_{1}^{\infty} f(x) \, dx \) diverges, then \( \int_{1}^{\infty} g(x) \, dx \) diverges. A S N

j) If \( P(x) \) is a cumulative distribution function, then \( P(10) > P(12) \). A S N