MATH 105  EXAM 1  October 8, 2004

NAME: ________________________________

While a final answer is important, you earn points for all the work leading to that answer, not just the answer itself. Show all your steps clearly so you will be eligible for the most partial credit. You may use a calculator, but no notes, books, or other students. Good luck!

1.) (10 pts.) Sketch the graph of a continuous function $f$ with all of the following properties:

i.) $f(0) = 2$

ii.) $f(x)$ is decreasing for $0 \leq x \leq 3$

iii.) $f(x)$ is increasing for $3 < x \leq 5$

iv.) $f(x)$ is decreasing for $x > 5$
2.) (10 pts.) The hours of daylight in Lewiston on the first day of each month of 2004 are given in the following table:

<table>
<thead>
<tr>
<th>Month:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours:</td>
<td>9</td>
<td>9.9</td>
<td>11.2</td>
<td>12.8</td>
<td>14.2</td>
<td>15.3</td>
<td>15.4</td>
<td>14.6</td>
<td>13.2</td>
<td>11.7</td>
<td>10.2</td>
<td>9.1</td>
</tr>
</tbody>
</table>

a.) What is the average change in daylight per month from June 1 to July 1?

b.) What is the average change in daylight per month from September 1 to December 1?

3.) (10 pts.) Compute the limit \( \lim_{x \to 0} \frac{x}{\sin 3x} \) using a table of values and/or a graph. (Your choice!)
4.) (10 pts.) Using the limit definition of the derivative, compute \( f'(x) \) if \( f(x) = \frac{1}{\sqrt{x}} \).
5.) (10 pts.) Sketch the graph of the derivative of the following function:

![Graph of a function](image)

6.) (10 pts.) Suppose \( P = f(t) \) is the population of Mexico in millions, where \( t \) is the number of years since 1980. Explain the meaning of the statements:

a.) \( f(6) = 86 \)

b.) \( f'(6) = 2 \)

c.) \( f^{-1}(95.5) = 16 \)
7.) (10 pts.) Is every continuous function also differentiable? If yes, explain why. If no, give an example to show why not.

8.) (10 pts.) Find the equation of the tangent line to \( g(x) = \frac{1}{2}(x^5 + 2x - 9) \) at \( x = 0 \).
9.) (10 pts.) Differentiate \( y = \frac{e^x}{x}, \frac{e^x}{x^2}, \) and \( \frac{e^x}{x^3}. \) Do you see a pattern emerging? What is it?

10.) (10 pts.) Fill in the table based on the graph shown below. Indicate in the appropriate boxes whether \( f, f', \) or \( f'' \) is positive (+), negative (−), or zero (0).

<table>
<thead>
<tr>
<th>Point</th>
<th>( f' )</th>
<th>( f'' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( C )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>