1. (12 points) Evaluate the following integrals exactly (without using the table of integrals):

   (a) \[ \int \sqrt{x} \ln x \, dx \]

   (b) \[ \int_{0}^{\pi/4} \sin^2(2\theta) \cos(2\theta) \, d\theta \]
2. (18 points) Evaluate the following integrals. In case of an improper integral, determine the convergence of the integral. Show clearly any limit computation you do. If the integral converges, find its value.

(a) \[ \int \frac{4x^3 - 4x^2 - 23x - 1}{x^2 - x - 6} \, dx \]

(b) \[ \int \frac{5}{t^2 + 6t + 11} \, dt \]
3. (6 points) Suppose a function $f(x)$ is increasing and concave down on the interval $[-1, 2]$. Put the following quantities in ascending order:

\[ \text{LEFT}(50), \text{RIGHT}(50), \text{MID}(50), \text{TRAP}(50), \int_{-1}^{2} f(x) \, dx \]

Explain briefly in words or with a sketch why your order is correct. What can you say with certainty about where SIMP(50) would fit into your list?
4. (6 points) For a function $g(x)$, $\int_{0}^{10} g(x) \, dx$ is approximated using the right rule. Suppose $\text{RIGHT}(5)=20.58$ and $\text{RIGHT}(15)=22.1$. Estimate the actual value of $\int_{0}^{10} g(x) \, dx$.

5. (8 points) Water flows into a storage tank at a rate of $r(t) = \frac{t}{\sqrt{3t^2 + 1}}$ ft$^3$/sec, where $t$ is the number of seconds since the water starts to flow in.

Find $\int_{0}^{4} r(t) \, dt$.

What does your answer represent?