1. Find \[
\begin{bmatrix}
1 & 3 & -5 & 6 \\
0 & 1 & -3 & 2 \\
2 & 5 & -7 & 10
\end{bmatrix}
\begin{bmatrix}
3 \\
1 \\
0 \\
2
\end{bmatrix}
= 3 \begin{bmatrix} 0 \\ 2 \\ -3 \\ -7 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 1 \\ 5 \\ -7 \end{bmatrix} + 0 \begin{bmatrix} -5 \\ -3 \\ -7 \\ 10 \end{bmatrix} + 2 \begin{bmatrix} 6 \\ 2 \\ 12 \\ 10 \end{bmatrix}
= \begin{bmatrix} 3 + 3 + 0 + 12 \\ 0 + 1 + 0 + 4 \\ 6 + 5 + 0 + 20 \\ 0 + 5 + 0 + 20 \end{bmatrix}
= \begin{bmatrix} 18 \\ 9 \\ 31 \end{bmatrix}
\]

2. Let \( A \) be the \( 3 \times 4 \) matrix from problem 1. Express all solutions of \( Ax = b \) in the form \( \mathbf{p} + \mathbf{v}_h \), where \( \mathbf{p} \) is a particular solution of \( Ax = b \) and \( \mathbf{v}_h \) represents all solutions to the corresponding homogeneous equation \( Ax = 0 \). Here, \( b = \begin{bmatrix} 70 \\ 20 \\ 120 \end{bmatrix} \).

Hint. \[
\begin{bmatrix}
1 & 0 & 4 & 0 & 10 \\
0 & 1 & -3 & 2 & 20 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\] is row equivalent to the augmented matrix you would set up to begin solving this problem.

(Note: it may help to write this:
\[
\begin{cases}
x_1 = 10 - 4x_3 \\
x_2 = 20 + 3x_3 - 2x_4 \\
x_3 \text{ is free} \\
x_4 \text{ is free}
\end{cases}
\]

One solution is \( \mathbf{p} = \begin{bmatrix} 10 \\ 20 \\ 0 \\ 0 \end{bmatrix} \) and \( \mathbf{v} = \begin{bmatrix} -4 \\ 3 \\ 1 \\ 0 \end{bmatrix} \), so \( \mathbf{b} = \mathbf{p} + \mathbf{v} \).

3. Let \( A \) and \( b \) be as in problem 2. Let \( a_1, a_2, a_3 \) and \( a_4 \) be the columns of \( A \). Use your answer to (2) to express \( b \) as a linear combination of \( a_1, a_2, a_3 \) and \( a_4 \) in 3 different ways, the first of which should be by setting all free variables to 0. Make clear your choices of values for the free variables. (See on-board example).

**LC 1:** We were told to use \( x_3 = x_4 = 0 \). This gives \( x_1 = 10 \) and \( x_2 = 20 \). Our LC is \[10a_1 + 20a_2 \] (or \[ 10a_1 + 20a_2 + 0a_3 + 0a_4 \]).

**LC 2:**
For LC2 & LC3, there are many possible answers. Commonly seen on students' quizzes were these examples:

- Choose \( x_3 = 1 \) and \( x_4 = 0 \). Get \[6a_1 + 23a_2 + a_3 \]
- \( x_3 = 1 \) and \( x_4 = 1 \). Get \[6a_1 + 21a_2 + a_3 + a_4 \]
- \( x_3 = 2 \) and \( x_4 = 2 \). Get \[2a_1 + 22a_2 + 2a_3 + 2a_4 \]
- Etc. (so \( x_1 = 10 - 8 = 2 \) and \( x_2 = 20 + (20 - 2) = 20 + 6 - 4 = 22 \))