Answer Key for Quiz 2 (section B)

1. If \( f(x) = \sqrt{2x + 1}, \) then \( f(x + h) = \sqrt{2(x + h) + 1}, \) so the limit we have to work out is

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{2x + 2h + 1} - \sqrt{2x + 1}}{h}.
\]

To do this we use the conjugate trick:

\[
f'(x) = \lim_{h \to 0} \frac{\sqrt{2x + 2h + 1} - \sqrt{2x + 1}}{h} \cdot \frac{\sqrt{2x + 2h + 1} + \sqrt{2x + 1}}{\sqrt{2x + 2h + 1} + \sqrt{2x + 1}}
\]

\[
= \lim_{h \to 0} \frac{2h}{\sqrt{2x + 2h + 1} + \sqrt{2x + 1}}
\]

\[
= \frac{2}{2\sqrt{2x + 1}}
\]

\[
= \frac{1}{\sqrt{2x + 1}}.
\]

So this function has the peculiar property that its derivative equals its reciprocal. In fact the only functions with this property are \( f(x) = \pm \sqrt{2x + a} \) for some constant \( a. \)

2. \( g(x) \) is perfectly flat (within the limits of my artistic ability) for \( x = B, \) \( x = D \) and \( x = F, \) so these are all the places where \( g'(x) = 0. \) \( g(x) \) is increasing for \( x < B \) and for \( D < x < F, \) so \( g'(x) \) is positive only for these values of \( x. \) \( g(x) \) is decreasing for \( B < x < D \) and for \( x > F, \) so \( g'(x) \) is negative for those values of \( x. \) \( g(x) \) is also almost flat for \( |x| \) large—it has a very small positive slope if \( x \) is a large negative number, and a very small negative slope if \( x \) is a large positive number. Therefore \( g'(x) \) must approach 0 from above as \( x \to -\infty \) and from below as \( x \to \infty. \) All this forces \( g(x) \) to have two peaks (local maximums) which are approximately at \( A \) and \( E, \) and two valleys (local minimums) which are approximately at \( C \) and \( G. \) Because \( g(x) \) appears to be an even function (symmetric about the \( y \)-axis), \( g'(x) \) will be an odd function (symmetric through the origin). So the graph of \( g'(x) \) must look about like this: