Consider the system of equations

\[
x_1 + 5x_2 + 3x_3 + 7x_4 = t
\]
\[
20x_1 + 100x_2 + 62x_3 + 146x_4 = w,
\]

where \( p \), \( t \), and \( w \) are three real numbers whose values we don't know.

1A. What is the augmented matrix for this system?

\[
\begin{bmatrix}
0 & 0 & 1 & 3 & p \\
1 & 5 & 3 & 7 & t \\
20 & 100 & 62 & 146 & w
\end{bmatrix}
\]

1B. By hand, find the matrix in RREF which is row equivalent to the answer in (1A). Show and label all your steps as we've done in class, for example, if you add 4 copies of row 5 to row 8, write \( r_8 \leftarrow r_8 + 4r_5 \); if you swap rows 4 and 5, write \( \text{swap } r_4 \text{ and } r_5 \), etc. (you don't need the quotes) Use steps that make the work easy; avoid fractions if possible. Note well that your column with \( p \), \( t \), and \( w \) will change as you change the coefficient side of the matrix.

\[
\begin{bmatrix}
0 & 0 & 1 & 3 & p \\
1 & 5 & 3 & 7 & t \\
20 & 100 & 62 & 146 & w
\end{bmatrix}
\]

1C. Use the answer to (1B) to determine what relationship \( p \), \( t \), and \( w \) must satisfy in order for this system to have a solution.

The original system (top of the page) is consistent

\[
w - 20t - 2p = 0
\]

this requires \( 8 \) more steps:

\[
\begin{bmatrix}
\text{(same)} & \begin{bmatrix} t - 3p \\ \rho \\ \text{(same)} & 0 \\ \text{(same)} & 0 \\
\end{bmatrix}
\end{bmatrix}
\]

I accepted this answer

however, if

\[ w - 20t - 2p \neq 0 \]

then to be in RREF this last entry must be \( 1 \) and the entries above it must be \( 0 \)'s:
2. Evaluate the following:

\[
\begin{bmatrix}
3 & 2 \\
5 & -1 \\
6 & 4
\end{bmatrix}
\begin{bmatrix}
4 \\
5
\end{bmatrix}
= 4 \begin{bmatrix}
3 \\
5 \\
6
\end{bmatrix}
+ 5 \begin{bmatrix}
2 \\
-1 \\
4
\end{bmatrix}
= \begin{bmatrix}
12 \\
20 \\
24
\end{bmatrix}
+ \begin{bmatrix}
10 \\
-5 \\
20
\end{bmatrix}
= \begin{bmatrix}
22 \\
15 \\
44
\end{bmatrix}
\]

3. Let \( A = \begin{bmatrix}
1 & 4 & 4 & 3 \\
3 & 12 & 13 & 11 \\
2 & 8 & 7 & 4
\end{bmatrix} \) and let \( b = \begin{bmatrix}
18 \\
61 \\
29
\end{bmatrix} \).

3A. Show how to write \( b \) as a linear combination of the columns of \( A \) (find all solutions and express them in terms of any free variables).

By calculator, the RREF of \( [A|b] \) is:

\[
\begin{bmatrix}
1 & 0 & 1 & 2 & 7 \\
0 & 0 & 1 & 2 & 7 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Which says:

\[
\begin{aligned}
b &= x_1 \begin{bmatrix}
1 \\
2 \\
8
\end{bmatrix}
+ x_2 \begin{bmatrix}
4 \\
12 \\
7
\end{bmatrix}
+ x_3 \begin{bmatrix}
13 \\
2 \\
1
\end{bmatrix}
+ x_4 \begin{bmatrix}
3 \\
4
\end{bmatrix} \\
\text{Where}
\end{aligned}
\]

\[
\begin{aligned}
x_1 &= -10 - 4x_2 + 5x_4 \\
x_3 &= 7 - 2x_4 \\
\text{and } x_2 \text{ and } x_4 \text{ are free}
\end{aligned}
\]

3B. Do the columns of \( A \) span \( \mathbb{R}^3 \)? Explain your answer.

\[\text{NO. There are 1's for which the last row of the RREF of } [A|b] \text{ will be } [0 0 0 0 1] \text{ instead of all 0's, i.e. the last row will lead to an inconsistency in solving } Ax = b \]

\[\text{Such 1's will not be in the span...}\]

\[\text{ALTERNATIVELY: Not every row of } A \text{ has a pivot position so by theorem 4, } A \text{'s columns cannot span } \mathbb{R}^3.\]