1.) Circle the appropriate word to complete each of the following statements correctly.

a) If the individual terms of a series tend to 0 ($\lim_{n \to \infty} a_n = 0$), then the series $\sum_{n=1}^{\infty} a_n$ will converge (always/sometimes/never).

b) If the individual terms of a series tend to 0.5 ($\lim_{n \to \infty} a_n = 0.5$), then the series $\sum_{n=1}^{\infty} a_n$ will converge (always/sometimes/never).

c) If the individual terms of an alternating series tend to 0 ($\lim_{n \to \infty} a_n = 0.5$), then the series $\sum_{n=1}^{\infty} a_n$ will converge (always/sometimes/never).

d) If the individual terms of a geometric series tend to 0 ($\lim_{n \to \infty} a_n = 0$), then the series $\sum_{n=1}^{\infty} a_n$ will converge (always/sometimes/never).

e) If the ratio of the terms of a series tends to 1 ($\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = 1$), then the series $\sum_{n=1}^{\infty} a_n$ will converge (always/sometimes/never).

f) If the ratio of the terms of a series tends to 0.5 ($\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = 0.5$), then the series $\sum_{n=1}^{\infty} a_n$ will converge (always/sometimes/never).

g) If a series has all positive terms, then it will converge to 0 (always/sometimes/never).

2.) Decide if each of the following converges or diverges. Give an explanation for your answers.

a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

b) $\sum_{n=1}^{\infty} \frac{(2n)!}{3^n(n!)^2}$
c) \[ \sum_{n=1}^{\infty} \left( \frac{1}{100} + \frac{1}{n^5} \right) \]

d) \[ \sum_{n=1}^{\infty} \frac{\sqrt{5n^8 + 4n^5}}{11n^6 + 13n^2} \]

e) \[ \sum_{n=1}^{\infty} \frac{(\ln(n))^2}{n} \]

3.) Compute the radius and interval (including endpoints) of convergence for \[ \sum_{n=1}^{\infty} \frac{(x + 3)^n}{(5^n)(2n)}. \]

4.) Find the complete Taylor series (in summation notation) for \( f(x) = \ln(1 - x) \) about \( x = 0 \).
5.) Let \( f(x) = x^3 \sin (-5x^2) \).

a) Write out the first 3 non-zero terms in the Taylor series about \( x = 0 \) for \( f(x) \).

b) Write out the complete series for \( f(x) \) in summation notation.

c) Write out the complete series for the function \( g(x) = 3x^2 \sin (-5x^2) - 10x^4 \cos (-5x^2) \) in summation notation.

6.) Find the general solution of the differential equation \( \frac{dy}{dx}(1 + x^3) = x^2 e^y \).

How could you check that your solution is correct?
7.) Use Euler’s Method with 3 steps to estimate \( y(3/4) \) if \( dy/dx = y - 4x \) and \( y(0) = 2 \).

8.) A colony of endangered sea otters has an annual birth rate of 10% and an annual death rate of 15%. In an attempt to sustain the colony, activists bring in otters from another region where the animals are plentiful. They do this at a rate of 50 otters per year.

   a) Write a DE whose solution is \( P(t) \), the otter population \( t \) years from now.

   b) Find any and all equilibrium solutions.

   c) Find the general solution of your DE.

   d) Find and sketch the particular solution if the current population is 400 otters.