1.) Consider the function \( f(x) = x^6 - 2x^3 \) on the interval \([-2,2]\).

a) Find the \( x \)-coordinate(s) of any and all roots of \( f \). These are the solutions of \( f(x) = 0 \). (No need for calculus yet, but this will help us sketch \( f \) later.)

b) Find the \( x \)- and \( y \)-coordinates of any and all local extrema and classify each as a local maximum or local minimum.

c) Find the \( x \)- and \( y \)-coordinates of any and all global extrema and classify each as a global maximum or global minimum.

d) Find the \( x \)-coordinate(s) of any and all inflection points.

e) Sketch \( f(x) \), labelling all the points you found above.
2. Your company is mass-producing a cylindrical container. The flat portion (top and bottom) costs 3 cents per square inch and the curved (lateral) portion costs 5 cents per square inch. If your budget is $9.00 per container, what dimensions give the largest volume?

area of circle = \( \pi r^2 \)  
lateral area of cylinder = \( 2\pi rh \)  
volume of cylinder \( \pi r^2h \)

3. Decide whether the Mean Value Theorem applies to each of the functions below. If it does apply, find the value of the constant \( c \) that the theorem specifies.

a) \( f(x) = \frac{1}{1 - \ln x} \) on \([1,3]\)

b) \( f(x) = x^3 + x \) on \([0,3]\)
4.) Circle the appropriate word to complete each of the following statements correctly.

a) If \( f'(3) = 0 \), then \( f \) has a local max or a local min at \( x = 3 \) (always/sometimes/never).

b) If \( f \) has a local max at \( x = 3 \), then \( f'(3) = 0 \) (always/sometimes/never).

c) If \( f'(3) = 0 \) and \( f''(3) = -2 \), then \( f \) has a local max at \( x = 3 \) (always/sometimes/never).

d) If \( f \) has a global max at \( x = 3 \), then \( f'(3) = 0 \) or \( f''(3) \) is undefined (always/sometimes/never).

e) If \( f''(3) = 0 \), then \( f \) has an inflection point at \( x = 3 \) (always/sometimes/never).

f) If \( f \) is continuous at \( x = 3 \), then \( f \) is differentiable at \( x = 3 \) (meaning \( f'(3) \) exists) (always/sometimes/never).

g) If \( f \) is differentiable at \( x = 3 \), then \( f \) is continuous at \( x = 3 \) (always/sometimes/never).

h) If \( \lim_{x \to a} \frac{f(x)}{g(x)} = 0 \), then \( \lim_{x \to a} \frac{f(x)}{g(x)} = 1 \) (always/sometimes/never).

i) If \( \lim_{x \to a} \frac{f(x)}{g(x)} = \infty \), then \( \lim_{x \to a} \frac{f(x)}{g(x)} = 1 \) (always/sometimes/never).

j) If \( f \) is continuous for \( 2 < x < 7 \), \( f \) has a global max and global min on that interval (always/sometimes/never).

5.) Water is leaking out of a tank at a decreasing rate. Find an overestimate and underestimate for the total amount that leaked out during these 8 minutes.

<table>
<thead>
<tr>
<th>time (min)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>rate (gal/min)</td>
<td>15</td>
<td>11</td>
<td>8</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

6.) Evaluate the following.

a) \( \int_1^7 \frac{3}{x} \, dx \)

b) \( \int (1 + 2x + 3x^2 + 4\sqrt{x} + \frac{1}{x^3} + 6^x) \, dx \)

c) \( \int_0^2 e^{3x} \, dx \)

d) \( \int \frac{7 - 3x^2}{5x^2} \, dx \)

e) \( \frac{d}{dx} \int_{x^4}^{39} \sin \sqrt{x} \, dx \)
7. The rate of change of a room’s temperature is \( r(t) = t^2 - 9 \) degrees per hour on the interval \([0, 4]\) hours. At \( t = 0 \), the temperature is 70 degrees. (Remember that \( r \) is the derivative of the temperature function.)
   a) When on this interval is the temperature rising? falling?

   b) What is the maximum temperature on this interval and when does it occur?

   c) What is the minimum temperature on this interval and when does it occur?

   d) What is the average rate of change of the temperature on this interval?

8.) An object is launched vertically into the air from ground level with an initial velocity of 160 feet per second. Gravity causes a downward acceleration of 32 ft/sec/sec. What is its velocity when it first reaches a height of 256 feet? When it next reaches this same height?