Math 105: Review for Final Exam, Part I

1. Consider the function \( f(x) = \frac{3}{5 - 2x} \).

   (a) Is this function continuous on the interval \((-\infty, \infty)\)? Explain.

   (b) Compute the average rate of change of \( f \) on \([2, 2.01]\).

   (c) Using the limit definition of the derivative, compute \( f'(x) \).

   (d) Find the equation of the tangent line to \( f \) at \( x = 2 \).

2. Given that \( f(0) = 2 \), \( g(0) = 3 \), \( f'(0) = 5 \), \( g'(0) = 7 \), and \( f'(3) = \pi \) compute the following.

   (a) \( h'(0) \) if \( h(z) = f(z)g(z) \)

   (b) \( j'(0) \) if \( j(z) = \frac{f(z)}{g(z)} \)

   (c) \( k'(0) \) if \( k(z) = f(g(z)) \)
3. (a) Find $\frac{dy}{dt}$ if $y = t^5 + e^5 + \frac{t}{5} + \frac{5}{t} + \frac{5}{\sqrt{t}} + \ln (5t) + \arctan (5t) + \ln(5) + \sin 5$.

(b) Find $\frac{dy}{dx}$ if $y = \sqrt[3]{x} \cos(7x^3)$.

(c) Find $\frac{dy}{dz}$ if $y = e^z + e^\pi \frac{\tan 4}{\tan 4 - 7z}$.

(d) Find $\frac{dy}{dr}$ if $y = \tan (e^{r^2 \arcsin(5r)})$.

(e) Find $\frac{dy}{dx}$ if $y^3 + yx^2 + x^2 = 3y^2$.

(f) Find $\frac{dy}{dx}$ if $y = (1 + x^6)^8x$.
4. Given the graph of \( f \), sketch a graph of \( f' \) and a graph of \( F \), an antiderivative of \( f \) such that \( F(0) = -1 \).

\[ \text{(Graphs of } f, f', \text{ and } F) \]

5. Shown below is a graph of \( f' \) on its entire domain. The graph is NOT \( f \).

At which \( x \)-value(s) (if any)
(a) does \( f \) have a stationary point?
(b) does \( f \) have a local max?
(c) does \( f \) have a local min?
(d) does \( f' \) have a stationary point?
(e) does \( f' \) have a local max?
(f) does \( f' \) have a local min?
(g) is \( f \) greatest?
(h) is \( f \) least?
(i) is \( f' \) greatest?
(j) is \( f' \) least?
(k) is \( f'' \) greatest?
(l) is \( f'' \) least?

On what interval(s) is
(a) \( f \) increasing?
(b) \( f \) decreasing?
(c) \( f' \) increasing?
(d) \( f' \) decreasing?
(e) \( f \) concave up?
(f) \( f \) concave down?

6. Solve the IVP \( y' = e^x - \sin x + 5 \) given that \( y(0) = 3 \).
7. Evaluate the following limits.

(a) \( \lim_{x \to \infty} \frac{x^2}{\ln x} \)

(b) \( \lim_{z \to 0} \frac{\sin (5z) - 5z}{z^3} \)

(c) \( \lim_{x \to 0} \frac{e^x - 1}{\cos x} \)

(d) \( \lim_{r \to 3} \frac{r^3 - 8}{r - 2} \)

8. Consider the function \( f(x) = x^6 - 2x^3 \) on the interval \([-2, 2]\).

(a) Find the \( x \)- and \( y \)-coordinates of any and all critical points and use calculus to classify each as a local maximum, local minimum, or neither.

(b) Find the \( x \)- and \( y \)-coordinates of any and all global extrema and classify each as a global maximum or global minimum.

(c) Find the \( x \)-coordinate(s) of any and all inflection points.