1. Suppose an economy has three producing sectors: agriculture, meats, and processed foods. The open sector consists of people who just consume ("eat") all these foods. The four sectors are thus \( A, M, P \) and \( E \), respectively. Suppose to produce one unit of output, \( A \) requires 0.22 units of its own output, and 0.13 units of \( M \) and 0.017 units of \( P \). Making one unit of \( M \) requires 0.19, 0.11, and 0.08 units of \( A, M \), and \( P \), resp., and a unit of \( P \) consumes 0.1, 0.07 and 0.3 units of \( A, M \), and \( P \), resp. The final demand by the \( E \) sector is 40, 30, and 80 units of \( A, M \), and \( P \), resp.

1A. Find the consumption matrix \( C \).

\[
C = \begin{bmatrix}
A & 0.22 & 0.11 & 0.17 \\
M & 0.13 & 0.11 & 0.07 \\
P & 0.10 & 0.08 & 0.3
\end{bmatrix}
\]

1B. Find \( I - C \) and write it here; store it as \([B]\) in your calculator.

\[
(I - C) = \begin{bmatrix}
0.78 & -0.19 & -0.10 \\
-0.10 & 0.89 & 0.07 \\
0.10 & -0.08 & 0.70
\end{bmatrix}
\]

\( I \) is the identity matrix; \( C \) is the consumption matrix. \( I - C \) is the matrix that describes the excess demand, and \( B \) is the \( n \times n \) matrix that describes the excess demand for the \( n \) sectors.

1C. Use your calculator to find \((I - C)^{-1}\) and write it here. (Just use \([B]\) and the \("-1\") key)

\[
(I - C)^{-1} = \begin{bmatrix}
1.363 & 0.311 & 0.226 \\
0.216 & 1.183 & 0.149 \\
0.219 & 0.180 & 1.498
\end{bmatrix}
\]

1D. Let \( d \) be the final demand vector. In terms of \( C \) and \( d \), what is the equation which we set up to find the production vector \( x \)?

\[
\hat{x} = C\hat{x} + d
\]

**Note:** a "during the quiz" request to set calculators to show THREE decimal places.

1E. Find the production vector \( x \). (Hint: put the final demand vector \( d \) into your calculator as a matrix (say \([D]\)) and do an appropriate matrix multiplication.

\[
(I - C)^{-1} \hat{d} = \begin{bmatrix}
B \\
D
\end{bmatrix} = \begin{bmatrix}
81.918 \\
56.087 \\
132.398
\end{bmatrix}
\]

1F. BONUS! It turns out that \((I_3 + C + C^2 + C^3 + C^4 + C^5)\) \( d \) is \( \begin{bmatrix}
81.154 \\
55.618 \\
131.592
\end{bmatrix}\). What is the connection between this fact and your work in (1A-1E)?

We mentioned in class that (given the right conditions on \( C \))

\[
\lim_{n \to \infty} \left( I_3 + C + C^2 + C^3 + \cdots + C^n \right) \hat{d} = (I - C)^{-1} \hat{d}
\]

so \((I_2 + C + C^2 + \cdots + C^5)\) \( d \) should be an approximation to \((I - C)^{-1} \hat{d} \) and indeed the vectors in 1E & 1F are close.
2. Let \( \mathbf{A} = \begin{bmatrix} 5 & 6 & -6 \\ 3 & 8 & -6 \\ 3 & 6 & -4 \end{bmatrix} \).

2A. It's a fact that \( \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \) is an eigenvector of \( \mathbf{A} \). What is the corresponding eigenvalue? (An easy calculation)

Multiply \( \mathbf{A} \) by \( \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \) to find \( \lambda \):

\[
\mathbf{A} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}, \text{ so } \lambda = 5
\]

2B. It's also true that \( \lambda = 2 \) is an eigenvalue of \( \mathbf{A} \). If possible, find matrices \( \mathbf{P} \) and \( \mathbf{D} \) that show \( \mathbf{A} \) is diagonalizable, or explain why \( \mathbf{A} \) is not diagonalizable. Show all your work.

Let's find a basis for the eigenspace of 2. (If the dimension is 2, great. Otherwise either 1) \( \dim(\text{eigenspace}(2)) = 2 \) or 2) there's another eigenvector or 3) \( \mathbf{A} \) won't be diagonalizable.)

\[
(\mathbf{A} - 2\mathbf{I}) = \begin{bmatrix} 3 & 6 & -6 \\ 3 & 6 & -6 \\ 3 & 6 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

And a basis for the nullspace of THIS matrix is \( \left\{ \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \right\} \), which is a basis for the eigenspace of \( \lambda = 2 \).

So \( \mathbf{A} \) is diagonalizable and \( \mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1} \)

Where \( \mathbf{P} = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \) and \( \mathbf{D} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \) (for example)