1. Suppose that \( \mathbf{v}_1 = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix} \), \( \mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \) and \( \mathbf{v}_3 = \begin{bmatrix} 1 \\ x \\ y \end{bmatrix} \); let \( \mathbf{s} = \begin{bmatrix} -11 \\ 18 \\ 51 \end{bmatrix} \).

1a. Explain why \( \mathbf{v}_1 \perp \mathbf{v}_2 \).

1b. Find a unit vector in the direction of \( \mathbf{v}_2 \).

1c. Find \( x \) and \( y \) which make \( \mathcal{B} = \{ \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \} \) an orthogonal basis of \( \mathbb{R}^3 \). (Use good linear algebra techniques; your answer will involve a RREF).

1d. Use the formulas developed in class for orthogonal bases to find \( \alpha_2 \) for which \( \mathbf{s} = \alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \alpha_3 \mathbf{v}_3 \). (You do not have to find \( \alpha_1 \) and \( \alpha_3 \).)

2. If \( A = \begin{bmatrix} 2 & 3 & 3 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 1 & 5 & -6 \\ 2 & 1 & 1 & 2 \end{bmatrix} \) then \( \text{RREF}(A) \) is \( R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \). Find a basis for each of the following. Write vectors horizontally where appropriate.

2a. \( \text{Col}(A) \)  
2b. \( \text{Col}(R) \)

2c. \( \text{Row}(A) \)  
2d. \( \text{Row}(R) \)

2e. Express \( r_3 \) (i.e., row 3) of \( A \) as a linear combination \( r_3 = xr_1 + yr_2 + zr_4 \) of the other three rows of \( A \). (Hint: you will be on familiar ground if you write the vectors vertically to solve the problem; find \( x \) \( y \) and \( z \). Or explain why there are no such scalars. Use good linear algebra methods.)