Name: ___________________________

Mathematics 106  
Exam II  
March 18, 2002

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You must show all work to receive credit. Calculators are permitted. Give exact answers ($\ln 5, e^3$) unless requested otherwise.
1. Carefully show whether each of the following converges or diverges.

   a) \( \sum_{n=1}^{\infty} \frac{7^n(n+3)!}{(2n)!} \)

   b) \( \sum_{n=1}^{\infty} \frac{\sqrt{2n^9} + 7n^{10}}{8n^7 + 7n^6 + 6n^5} \)

   c) \( \sum_{n=1}^{\infty} \frac{1}{n^3} + \frac{1}{10} \)

2. Find the interval and radius of convergence of the power series \( \sum_{n=1}^{\infty} \frac{(x-5)^n}{2^n n^3} \). Explain how you determined its behavior at the endpoints.
3. a) Use a second degree Taylor polynomial to estimate $\sqrt{8.2}$. Give your answer as a simplified fraction.

b) What is the maximum possible error in your estimate? Simplify as much as possible.

4. a) Write out the first three non-zero terms in the Taylor series for $f(x) = x^{2002} \sin(3x)$ about $x = 0$.

b) Compute the $2007^{th}$ derivative of this same function at $x = 0$.

c) Evaluate $\lim_{x \to 0} \frac{\cos(2x^2) - 1 + 2x^2}{13x^4}$.

d) By ignoring terms of order three and higher, use Taylor series to estimate a solution to the equation $\sin(4x) + \cos(5x) = e^{ix}$ near $x = 0$. (This equation has an exact solution at $x = 0$, but you should estimate another solution near $x = 0$.)
5. You have started taking 800 mg of a drug each day at 8:00 am. In any 24-hour period, 5% of the drug in your bloodstream is eliminated from your body.
   a) How much (exactly) of the drug will be in your body immediately after the 8th dose? Simplify your answer as much as possible, but don’t just give a decimal approximation.

b) If you keep taking this drug for many years, will the amount in your body immediately after each dose level off? If so, what will this level be? If not, explain why not.

6. Carefully show whether or not \( \sum_{n=1}^{\infty} \frac{1}{n^3} \) converges. If it does converge, provide a good upper bound on its sum. In this problem only, you may NOT take for granted any facts we may have proved in class about the convergence of \( \sum_{n=1}^{\infty} \frac{1}{n^p} \). A sketch will probably be useful in your explanation.
7. Suppose that you use a 50th degree Taylor polynomial centered at $x = 0$ to estimate $e$ (that is, $e^1$).
   a) What is the maximum possible error in your estimate? You may use the fact that $e < 3$.
   b) What degree polynomial must you use in order to estimate $e$ to within $1/1,000,000$ of its true value? You may again use the fact that $e < 3$.

8. Circle the appropriate letter to indicate if each statement is true always (A), sometimes (S), or never (N).
   a) If the ratio in the ratio test is 1, the series converges. A S N
   b) If the terms of a series approach 0, the series converges. A S N
   c) If the terms of a geometric series approach 0, the series converges. A S N
   d) If the ratio in the ratio test is 0.5, the series converges to 0.5. A S N
   e) If the terms of a series alternate and approach 0, the series converges. A S N