(5) I. Give an example of vector field on $\mathbb{R}^3$.

(10) II. Find the equation of the tangent plane at the point $(0, 1, 1)$ to the surface with equation:

$$x^3 - 5y + 6yz^2 = 1.$$
(15) III. Suppose \( f(x, y, z) = xy^2 + x^2 y - z - 5x \) and \( \mathbf{a} = (1, 1, 1) \).

A. \( \nabla f(x, y, z) = \)

B. \( \nabla f(a) = \)

C. The directional derivative of \( f \) at \( \mathbf{a} \) in the direction parallel to the line \( \mathbf{x}(t) = (t + 1, 3t + 2, 2t + 3) \) is
(10) IV. Calculate the second-degree Taylor polynomial centered at \( a = (1, 2) \) of the function \( f : \mathbb{R}^2 \rightarrow \mathbb{R} \) with formula

\[
f(x, y) = xy + x^3.
\]

(10)V. If \( f(x, y) = x^3 + y^2 \), give the Hessian form for \( f \) at \((1, 0)\).
(10)VI. For the vector field \( \mathbf{F} = (xyz, xy^2 + z, 7) \)

A) \( \text{div} (\mathbf{F}) = \)

B) \( \text{curl} (\mathbf{F}) = \)

(10)VII. Find all critical points of \( f(x, y) = x^2 + y^2 - 4x \). Use the Second Derivative Test to determine whether each critical point is a local minimum, a local maximum, or neither.
VIII. Suppose you learn that the Jacobian of a function $f$ at $a$ is the matrix

$$
\begin{bmatrix}
1 & 3 & 2 \\
0 & 1 & 4
\end{bmatrix}
$$

Give a formula for $(Df(a))(x)$ using no more than the symbols $s, t, u, v, w, x, y, z,$ commas, and parentheses. Note: You don't have to use all of the symbols!

IX. Suppose $f(x, y, z) = (y, z, x)$ and $g(s, t) = (st, s - t, s + t)$ Use the chain rule to find the derivative of $f \circ g$ at the point $(3, 2)$. 