1. (a) Does $\sum_{n=1}^{\infty} \frac{1}{e^n}$ converge? If so, what does it converge to? Explain.

(b) Does $\int_{1}^{\infty} \frac{dx}{e^x + 3}$ converge? Explain. (If it converges, you may receive extra credit if you can say something about what it converges to.)
2. (i) Evaluate \( \int x \cos(x^2) \, dx \).

(ii) Using your answer to (i), integrate \( \int \cos(x^2) \, dx \) by parts with \( dv = x \cos(x^2) \, dx \) and \( u = \frac{1}{x} \).

(iii) Based on your answer to (ii), do you think \( \int_1^\infty \cos(x^2) \, dx \) converges, or do you think it diverges? Explain. (Hint: how large can \( \sin \theta \) be?)
3. Consider

\[ \int_1^\infty \frac{dx}{(x + 1)(x + 4)} \quad \text{and} \quad \sum_{n=1}^\infty \frac{1}{(n + 1)(n + 4)} \]

Do they converge, or do they diverge? Can you find the exact value of either or both? Explain.
4. (a) In what sense is it true that \( \sum_{n=1}^{\infty} \frac{1}{n} \) “is the smallest infinite series that diverges”?

(b) Evaluate \( \int \frac{dx}{x \ln x} \).

(c) Does \( \sum_{n=3}^{\infty} \frac{1}{n \ln n} \) converge, or does it diverge? Explain. What does your answer say about part (a)?