Please show all your work in order to receive partial credit.

1. Consider $f(x) = \begin{cases} 
x^2 + 1, & \text{if } x < 0 \\
1, & \text{if } x = 0 \\
\cos x, & \text{if } x > 0 
\end{cases}$

(a) Does $f(0)$ exist? If so, what is it?

$$f(0) = 1$$

(b) What is $\lim_{x \to 0^-} f(x)$?

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2 + 1 = 1$$

(c) What is $\lim_{x \to 0^+} f(x)$?

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \cos x = 1$$

(d) Is $f$ continuous at $x = 0$? Why?

$f$ is continuous at $x = 0$ because $\lim_{x \to 0} f(x) = 1 = f(0)$

(e) Does $f'(0)$ exist? Why?

Using a graphical approach we can zoom in on the graph of $f$ near $x = 0$.

From (d) we know the function is continuous. Close inspection of the graph reveals no sharp corners or vertical tangents at $x = 0$. More specifically, we can see that as $x$ approaches zero from both the left and right that the slope of the function is zero. Therefore, $f'(0) = 0$.

2. Find derivatives for the following functions.

(a) $f(x) = 3x^4 + x^2 - x + 2$

$$f'(x) = 12x^3 + 2x - 1$$

(b) $f(x) = \frac{x^3 + \sqrt{x} - 1}{x^2}$

First rewrite $f(x) = \frac{x^3}{x^2} + \frac{x^{1/2}}{x^2} - \frac{1}{x^2} = x + x^{-3/2} - x^{-2}$

So $f'(x) = 1 - \frac{3}{2}x^{-5/2} - (-2x^{-3}) = 1 - \frac{3}{2\sqrt{x^5}} + \frac{2}{x^3}$

(c) $f(x) = x^\pi + e^{\sqrt{x}}$

$$f'(x) = \pi x^{\pi - 1}$$ (Note: $e^{\sqrt{x}}$ is a constant so its derivative is zero)