Answer Key for Quiz 9 (section B)

1(a) We find the equilibrium solution of \( \frac{dy}{dx} = 3y - 6 \) by setting the right side equal to zero, which gives \( y = 2 \). This is a solution because it makes both sides of the differential equation equal to zero.

1(b) Separate variables and integrate:

\[
\int \frac{dy}{y-2} = \int 3 \, dx \implies \ln |y-2| = 3x + C,
\]

where \( C \) is an arbitrary constant. To find it we plug in \( x = 0 \) and \( y = 3 \), which gives

\[
\ln |3-2| = 0 + C \implies C = \ln 1 = 0.
\]

Moreover, since \( y \) is above the equilibrium solution when \( x = 0 \) it has to stay there for all \( x \), so \( y - 2 \) is always positive and therefore we can drop the absolute values, which gives \( \ln(y - 2) = 3x \). Exponentiating both sides we get

\[
e^{\ln(y-2)} = e^{3x} \implies y - 2 = e^{3x} \implies y = e^{3x} + 2.
\]

2. Separate variables and integrate:

\[
\int \frac{dy}{y^2 + 1} = \int 3(x^2 + 1) \, dx = \int (3x^2 + 3) \, dx.
\]

On the right we just have to integrate power functions, and we can look up the integral on the left (#24 with \( a = 1 \)). Then we have

\[
\arctan y = x^3 + 3x + C.
\]

To find \( C \) plug in \( x = 0 \) and \( y = 0 \), which gives

\[
\arctan 0 = 0 + C \implies C = \arctan 0 = 0.
\]

Therefore

\[
\arctan y = x^3 + 3x,
\]

and we can solve this for \( y \) by taking the tangent of both sides:

\[
\tan (\arctan y) = \tan (x^3 + 3x) \implies y = \tan (x^3 + 3x).
\]