Answer Key for Quiz 6

1. Separate variables to get \( \frac{dy}{y} = e^x \, dx \), and integrate both sides:

\[
\int \frac{dy}{y} = \int e^x \, dx, \quad \text{which implies} \quad \ln |y| = e^x + C.
\]

Plugging in \( y = 1 \) and \( x = 0 \) we get

\[
\ln |1| = e^0 + C, \quad \text{which implies} \quad 0 = 1 + C.
\]

Therefore \( C = -1 \); moreover, since we have one positive value of \( y \) in the term \( \ln |y| \) then \( y \) must always be positive there, so now we know that

\[
\ln y = e^x - 1.
\]

Finally, we solve this for \( y \) by exponentiating both sides:

\[
e^{\ln y} = e^{e^x - 1}, \quad \text{which implies} \quad y = e^{e^x - 1} \quad \text{since} \quad e^{\ln y} = y.
\]

2. Separate variables to get \( (y + 1) \, dy = (x + 1) \, dx \) and integrate both sides:

\[
\int (y + 1) \, dy = \int (x + 1) \, dx, \quad \text{which implies} \quad \frac{y^2}{2} + y = \frac{x^2}{2} + x + C.
\]

Plugging in \( y = -2 \) and \( x = 0 \) we get

\[
\frac{(-2)^2}{2} - 2 = \frac{0^2}{2} + 0 + C, \quad \text{which implies} \quad 2 - 2 = C,
\]

so \( C = 0 \). Putting this in and multiplying by 2 we have

\[
y^2 + 2y = x^2 + 2x,
\]

and it remains to solve this for \( y \) if possible. This is a little tricky, but here are two possible methods: add 1 to both sides to get

\[
y^2 + 2y + 1 = x^2 + 2x + 1, \quad \text{which implies} \quad (y + 1)^2 = (x + 1)^2.
\]

Taking square roots we have \( y + 1 = \pm (x + 1) \), so

\[
either \quad y + 1 = x + 1 \quad or \quad y + 1 = -1 - x.
\]

Since \( y = -2 \) when \( x = 0 \), the second one must be right, so we finally have \( y = -x - 2 \). Another method is to rewrite \( y^2 + 2y = x^2 + 2x \) as

\[
y^2 - x^2 + 2y - 2x = 0 = (y - x)(y + x) + 2(y - x) = (y - x)(y + x + 2).
\]

Therefore

\[
either \quad y - x = 0 \quad or \quad y + x + 2 = 0;
\]

in other words,

\[
either \quad y = x \quad or \quad y = -x - 2.
\]

Since \( y = -2 \) when \( x = 0 \) the second one must be right, and we again conclude that \( y = -x - 2 \).