Introduction to models:
Stocks and Flows

Models are conceptual tools by which we come to understand the implications of our theories about the world. Models come in many varieties and can be classified in many different ways. We often think about: verbal models; conceptual or graphical models; analytical mathematical models; simulation models.

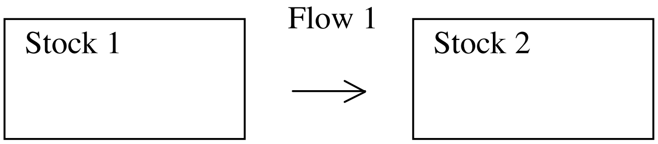
It is important to begin by embracing a key idea – that models are always wrong. A model is, be definition, a simplification, an idealization, a representation. We construct models because they help us think about complex things, but don't lose track of why we construct models or the fact that models can't explain our world perfectly. Inflexible minds can often be identified by their refusal to accept the limitations of models.

Many models can be recast in terms of an effort to track changes in the location or abundance (the flow) of one or more "things" (the stock).

Examples:

Fisheries Models (Fish populations, fishermen, dollars)
Economic models (dollars, goods)
Ecosystem simulation models (carbon, oxygen, water, nutrients)
Community models (individual plants or plant species)
Management ("employee morale", dollars)

We (and others) have found the use of stock and flow models to be particularly helpful in thinking about issues in environmental science. Stock and flow models are constructed from stocks and flows. They can be built as formal mathematical models (in our work we have found the program Stella to be particularly useful in developing mathematical models for a mathematically unsophisticated audience). However, they can also be used to clarify thinking in non-mathematical models. We generally diagram our models using boxes to represent stocks and arrows for flows.



The basic approach to thinking about stock and flow models is quite simple.

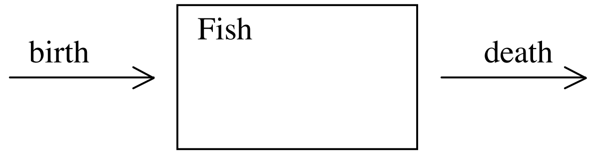
1. *Identify the major stocks in your model*
2. *Identify major flows between stocks*

   You have now constructed the basic structure of your model. From here on out, the task is to figure out ways to estimate the magnitude of various stocks and flows, and how they are interrelated.

3. *Select ways to describe the size of the flows between stocks*
   - Constant?
   - Dependent on the source stock?
   - Dependent on the receiving stock?
   - Dependent on another flow elsewhere in the model?
4. *Examine steady-state conditions*
   - Write out the steady-state equation for each stock
   - If your system is a closed system, or a conservation law applies, you may want to calculate the total amount of material or energy in the system
   - Collect available data on stocks, flows, or parameters
   - Solve for steady-state conditions
   - Determine Residence Times
5. *Examine transient dynamics*
   - Collect available data stocks, flows, or parameters
   - Draw a STELLA Model
   - Run simulation
   - You might need calculus, linear algebra, and differential equations to solve the model analytically. Stella makes this type of thinking more accessible.

We begin by looking at simple fisheries models. We recognize that there are many possible ways to model population.

growth = births-deaths



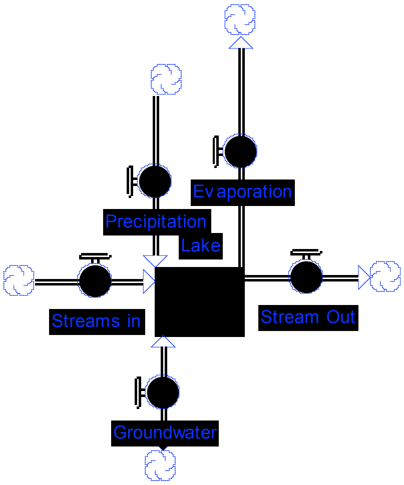
This is a one-stock model and the basic structure is simple. There is one inflow and one outflow.

But, there are many alternative functions possible. For example, we can think about both births and deaths in the following ways.

Births = constant (e.g. territorial reef fishes with planktonic larvae
Births = K * N (births are
Births = (r(K-N/K) (logistic growth)
Deaths = Constant?
Deaths = K * N (random mortality)
Deaths = K * N - Harvest
Deaths = K * N - E*N (harvest per unit effort)
Deaths = K*N^2 (canibalism)

Here are a couple of exercises for you to try on your own.

1. Construct an Eastern Lake Water Budget
One stock model -- the lake



INFLOWS:

Streams_in = Stream_Inflow
Groundwater = Groundwater_Rate
Precipitation = Lake_Area*Precipitation_per_Year*10^6/10

OUTFLOWS:

Stream_Out = Stream_Outflow
Evaporation = Evaporation_Rate*Lake_Area*10^6/10

Parameters (These data are modified from a real Michigan lake....)

Lake_Area = 0.45 km^2
Mean_Depth = 5 m
Groundwater_Rate = 175000 m^3/yr
Precipitation_per_Year = 100 cm/yr
Stream_Inflow = 250000 m^3/yr
Evaporation_Rate = 105 cm/yr

Stream_Outflow = 450000 m^3/yr
this corresponds to about 1/2 a cubic foot per second. We are talking about a pretty small stream here.... This is a stream that would fill up a bucket every second or two. It's about the flow of 25 garden hoses.
This also corresponds to a drainage area of perhaps a square mile or. That's only a bit bigger than the lake itself.
Is it in steady-state?

Would you expect a lake like this one to be in steady-state?

Do Inflows Equal Outflows?

Yes. (I confess, I set it up that way).

Details

Precipitation_per_Year	100 cm/yr *(1m/100cm)* 0.45 km^2 * (1000m/km)^2	450000m^3/yr
Stream_Inflow		250000 m^3/yr
Groundwater_Rate		175000 m^3/yr
Total Inputs		875000
Evaporation_Rate	105 cm/yr*(1m/100cm)* 0.45 km^2 * (1000m/km)^2	472500 m/yr
Stream_Outflow		402500 m^3/yr
Total Outputs		875000

Residence Time

Volume / Total inflows

Volume:

0.45 km^2 x (1000 m/km)^2 x 5 m = 2250000 cubic meters

Total inflows

175000 m^3, + 100cm * (0.45m/100cm) * 1 km^2 * (1000m/km)^2 +250000m^3 = 875000 cubic meters

Residence Time

2.25/0.875=2.6 years.
Notice that in this model, the dominant input were direct precipitation, with stream flow and groundwater each contributing about half of the remainder. That suggests a lake in a small watershed, although I actually don't have the data on that..

Since this is at steady state, this equals volume / outflows

2. Now consider a western lake model

Two Questions

1. Would a similar western lake need to occur in a smaller or larger watershed?

2. Would the residence time of water in the lake be larger or smaller than in the eastern lake?

Lets make a few basic assumptions, and think about how we build a lake in the Western United States

Some assumptions

Drop precipitation to 25cm/yr -- this is typical shortgrass prairie to semi-desert.
Double the evaporation rate (this is unrealistically high, even for most of the western U.S. Maybe this is more desert than prairie).
Assume groundwater leaves the lake at low quantities (50,000 m^3/yr) (inflow in Michigan lake was influences by springs)
Maintain a stream outflow the same as in the eastern lake model

Western Lake Model (Details)

Parameters:

Lake_Area = 0.45km^2
Evaporation_Rate = 200 cm/yr
Groundwater_Rate = -50000m^3/yr
Precipitation_per_Year = 25 cm/yr
Salt_Concentration_in_Stream = 0.001 (= 1 ppt or 0.1%)
Stream_Inflow = 1287500 m^3/yr
Stream_Outflow = 162500 m^3/yr
Stream_Outflow = 450000 m^3/yr

Details

Precipitation_per_Year	0.25 cm/yr *(1m/100cm)* 0.45 km^2 * (1000m/km)^2	112,500m^3/yr
Stream_Inflow		1287500 m^3/yr
Total Inputs		1,400,000
Evaporation_Rate	200 cm/yr*(1m/100cm)* 0.45 km^2 * (1000m/km)^2	900,000 m/yr
Stream_Outflow	(Assumed Constant)	450,000 m^3/yr
Groundwater_Rate		50,000 m^3/yr
Total Outputs		1,400,000

Residence Time

Volume / Total inflows

Volume:

0.45 km^2 x (1000 m/km)^2 x 5 m = 2250000 cubic meters

Total inflows

1,400,000 cubic meters

Residence Time

2.25/1.4 = 1.61 years.

Notice that in this model, the dominant input was STREAM flow, not precipitation

> *Even if both landscapes yielded the same amount of water, on an annual basis, you would need to have a larger watershed.*
>
> *Dryer western landscape yields substantially less water per unit area, so you will need a significantly larger watershed to support a lake with such a large outflow.*
>
> *Many actual western lakes and ponds do not have significant outflows.*
>
> *Many also exist episodically, and grow and shrink based on recent precipitation patterns.*
>
> *An annual model may be less informative in the western US.*

Which lake is more likely to become saline?

> *Add a Second Stock to the Western Lake model*
>
> *Two stock model*
>
> *Stocks are NOT connected by flows, but they are linked*

3. Here is a STELLA model example
   Soda machine model

Interest focuses on the probability of "running out" of sodas and thus missing sales. Refilling is episodic. If sales average about 2 sodas an hour, and the soda machine holds a maximum of 200 sodas, how often should you check to be certain that you never miss a sale because of an empty machine?

If it costs $18 to refill a machine, regardless of the amount that needs to be replaced, and you make $.10 on each soda sale (excluding the cost of refilling the machine), and the machine holds a maximum of 200 sodas, how frequently should you check to maximize your profits?
Key idea here is to realize that, since there is a fixed cost to refill, you are always better off letting the machine empty. However, to maximize profits, you do not want it to sit empty any longer than necessary. Thus we again find we want to refill every 100 hours, or about every four days.
Back of the envelope question

> *About how often should the soda machine in a site near you be refilled to maximize profits?*