Foundations of Physics, Vol. 12, No. 1, 1982

Experimental Verification of an Aharonov–Bohm Effect in Rotating Reference Frames

Mark D. Semon[1]

Received January 26, 1981

A thought experiment is reviewed which shows two things. First, in a region of a rotating frame that is not simply connected, the inertial forces can be canceled without completely canceling the inertial vector potential (whose curl determines the Coriolis force); second, the presence of this uncanceled potential can be detected in a quantum interference experiment. It is then argued that the thought experiment was realized in an earlier experiment involving a rotating superconductor, and that the experimental results confirm the theoretical prediction. In this way, the first experimental verification of a physical effect due to a nonelectromagnetic potential in a force-free region is established. An analogous experiment for the gravitational vector potential is also discussed. Finally, it is pointed out that the close connection between electromagnetic and inertial vector potentials provides an intuitive way to make predictions about rotating superconductors.

1. INTRODUCTION

The Aharonov–Bohm effect[1] has been discussed extensively in the literature.[2–5] As originally formulated, it showed that electromagnetic potentials can affect charged particles moving through regions in which the electromagnetic force is zero. This effect has been verified experimentally in the case of the electromagnetic vector potential.[12] More generally, the expression "Aharonov–Bohm effect" can be used to mean an effect caused by a potential in a force-free region. Although such effects are predicted to occur for gauge fields,[6,7] the gravitational field,[6,8] and for Yang–Mills fields,[6,9] in none of these cases (nor in the case of the electromagnetic scalar potential) has there been any experimental verifications.

[1] Department of Physics and Astronomy, Bates College, Lewiston, Maine 04240.

0015-9018/82/0100-0049$03.00/0

825/12/1-4

In 1973 Aharonov and Carmi[10] showed that there is also an Aharonov–Bohm effect for the inertial field. That is, they showed how, in a region of a rotating frame that is not simply connected, the inertial forces can be canceled without completely canceling the inertial vector potential (whose curl determines the Coriolis force). This uncanceled potential was then predicted to shift an interference pattern, and, thus, to have the same physical significance as the electromagnetic vector potential. In this paper we argue that an experiment performed in 1965 by Zimmerman and Mercereau[11] is essentially the one suggested by Aharonov and Carmi, and that the experimental results confirm the theoretical prediction. Thus, the experiment of Zimmerman and Mercereau is shown to be the first experimental verification of an Aharonov–Bohm effect in a nonelectromagnetic field.

Because of the close connection between inertial and gravitational fields, one expects to find a gravitational analog of the experiment of Zimmerman and Mercereau. A specific design for such an experiment is discussed. Finally, the Aharonov–Bohm effect for an inertial field is shown to be just one example of a more general connection between electromagnetic and inertial vector potentials, and this connection is shown to provide an intuitive way of making predictions about rotating superconductors.

2. THE THOUGHT EXPERIMENT OF AHARONOV AND CARMI

In this section we review a thought experiment of Aharonov and Carmi that predicts an Aharonov–Bohm effect for the inertial field. Although a similar derivation has appeared before,[5,10] additional details are presented here which fill in some previous gaps, and which should make the argument more convincing to some readers. Whenever possible, references are made to previous work.

Consider a ring of inner radius r_1 and outer radius r_2 rotating with a constant angular speed ω about an axis through $r = 0$ and perpendicular to the ring. An observer rotating with the ring will observe particles of mass m subject to Coriolis and centrifugal forces. By measuring these forces on various test particles, the rotating observer can assign an $\boldsymbol{\omega}$ vector to every point in the rotating frame, thus, defining an $\boldsymbol{\omega}$ field which, in the present case, is found to be uniform. Assigning an $\boldsymbol{\omega}$ field in this way is analogous to the assignment of $\mathbf{E}$ and $\mathbf{B}$ fields in an inertial frame. Since the $\boldsymbol{\omega}$ field is uniform

$$\nabla \cdot \boldsymbol{\omega} = 0 \tag{1}$$

which implies that there exists an *inertial* vector potential **a** such that

$$2\boldsymbol{\omega} = \nabla \times \mathbf{a} \tag{2}$$

The Hamiltonian for a mass m relative to an observer rotating with the disk can be written as[5]

$$H = \frac{(\mathbf{P} - m\mathbf{a})^2}{2m} + mV \tag{3}$$

where V is the centrifugal potential and **P** the canonical momentum

$$\mathbf{P} = m\mathbf{v} + m\mathbf{a}. \tag{4}$$

Next, suppose that **E** and **B** fields are introduced whose Lorentz force cancels the inertial forces in the region $r_1 < r < r_2$, and which are zero for $r < r_1$. The required fields are[2]

$$\mathbf{B} = -\left(\frac{2m}{q}\right)\boldsymbol{\omega} \tag{5}$$

$$\mathbf{E} = \frac{m}{q}\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \tag{6}$$

in the region $r_1 < r < r_2$. Of course the cancellation of the Lorentz and inertial forces on the ring will occur only for particles with the same charge to mass ratio q/m, so we assume these are the only test particles available to the rotating observer.

For test particles in arbitrary **E** and **B** fields, the Hamiltonian relative to an observer in a rotating frame is

$$H = \frac{(\mathbf{P} - m\mathbf{a} - q\mathbf{A})^2}{2m} + q\phi + mV \tag{7}$$

where **A** and ϕ are the electromagnetic vector and scalar potentials. In our particular case, two simplifications occur. First, at any given point on the ring (between r_1 and r_2) the cancellation of the electric and centrifugal forces implies that the zero of each scalar potential can be chosen such that $-q\phi = mV$. On the other hand, while the cancellation of the magnetic and Coriolis forces implies

$$q\nabla \times \mathbf{A} = -m\nabla \times \mathbf{a} \tag{8}$$

[2] Here and in what follows, we ignore effects of order v/c and higher.

it does *not* imply that $q\mathbf{A} = -m\mathbf{a}$ everywhere on the ring. To see this, we note that if we integrate $q\mathbf{A}$ around a path between r_1 and r_2 that encloses an area σ, we find (using Stoke's theorem) that

$$q \oint \mathbf{A} \cdot \mathbf{dl} = q \int \mathbf{B} \cdot \hat{n}\, d\sigma = -2m\omega(\sigma - \pi r_1^2) \tag{9}$$

since the $\mathbf{B}$ field of Eq. (5) is only nonzero for $r > r_1$. Integrating $m\mathbf{a}$ along the same curve, we find

$$m \oint \mathbf{a} \cdot \mathbf{dl} = 2m \int \boldsymbol{\omega} \cdot \hat{n}\, d\sigma = 2m\omega\sigma \tag{10}$$

This means that for any closed path between r_1 and r_2

$$\oint (q\mathbf{A} + m\mathbf{a}) \cdot \mathbf{dl} = 2m\omega\pi r_1^2 \tag{11}$$

and, thus, $q\mathbf{A}$ cannot equal $-m\mathbf{a}$ everywhere on the ring.

To see that the uncanceled vector potential is inertial and not electromagnetic, we note that Eq. (10) can be written

$$m \oint \mathbf{a} \cdot \mathbf{dl} = 2m\omega(\sigma - \pi r_1^2) + 2m\omega\pi r_1^2 \tag{12}$$

The first term on the right-hand side is the $\boldsymbol{\omega}$ flux through the area on the ring between the boundary of the integration curve and r_1, while the second term is the $\boldsymbol{\omega}$ flux through the area πr_1^2. Associated with each of these fluxes we can define inertial vector potentials $\mathbf{a}_1$ and $\mathbf{a}_2$ such that $\mathbf{a} = \mathbf{a}_1 + \mathbf{a}_2$,

$$m \oint \mathbf{a}_1 \cdot \mathbf{dl} = 2m\omega(\sigma - \pi r_1^2) \tag{13}$$

and

$$m \oint \mathbf{a}_2 \cdot \mathbf{dl} = 2m\omega\pi r_1^2 \tag{14}$$

Adding Eq. (12) to Eq. (9) shows that the uncanceled vector potential on the ring is the inertial vector potential $\mathbf{a}_2$, which is associated with the $\boldsymbol{\omega}$ flux through the region $r < r_1$. In other words, to the rotating observer, the region $r_1 < r < r_2$ is force-free, but contains a nonzero inertial vector potential $\mathbf{a}_2$ whose line integral around any closed curve on the ring is given by Eq. (14). Thus, the Hamiltonian of Eq. (7) becomes $H = (\mathbf{P} - m\mathbf{a}_2)^2/2m$, with $\nabla \times \mathbf{a}_2 = 0$ when $r_1 < r < r_2$, which is equivalent to the Hamiltonian that leads to the Aharonov–Bohm effect.[1,5]

Imagine building a two-slit interference experiment on the rotating ring such that the circular hole $r < r_1$ lies between the two slits, and behind the central barrier. Following the arguments of Aharonov and Bohm, we would predict a phase shift in the interference pattern of

$$\Phi = \left(\frac{m}{\hbar}\right) \oint \mathbf{a}_2 \cdot \mathbf{dl} = 2m\omega\pi r_1^2/\hbar \tag{15}$$

That is, if the two-slit interference experiment is first performed on the ring when it is not rotating and no external fields are present, an interference pattern will be observed. If the ring is then given a constant angular velocity $\boldsymbol{\omega}$ and if the canceling **E** and **B** fields of Eq. (5) and (6) are introduced, the phase of the interference pattern will shift by the amount given in Eq. (15).

Thus, the inertial vector potential is predicted to affect particles passing through a nonsimply connected force-free region. This is precisely the Aharonov–Bohm effect for the inertial field, and shows that the integral (modulo 2π)[3,9] of an inertial vector potential around a closed path has physical significance.

3. EXPERIMENTAL VERIFICATION

The Aharonov–Bohm effect for the inertial field is quite similar to the Aharonov–Bohm effect for the electromagnetic field: in the electromagnetic case, particles move through a force-free region with $\mathbf{A} \neq 0$ which is not simply connected and which encloses a **B** flux, while in the inertial case, particles move through a force-free region with $\mathbf{a} \neq 0$ which is not simply connected and which encloses an $\boldsymbol{\omega}$ flux. Thus, in order to find a real experiment verifying an Aharonov–Bohm effect for the inertial field it is natural to examine experiments performed in the electromagnetic case to see if it is possible to do analogous experiments with **B** replaced by $\boldsymbol{\omega}$ and **A** replaced by **a**.

One of the most convincing experiments verifying the Aharonov–Bohm effect for the electromagnetic field is the one performed by Jaklevic *et al.*[12] using a Josephson-junction interferometer (i.e., a SQUID). In this case a **B** field was introduced into the nonconducting region between the two Josephson junctions, and the maximum superconducting current was observed to be periodic as the magnetic flux was varied. This confirmed the Aharonov–Bohm effect for the electromagnetic vector potential, and showed conclusively that it is not due to a magnetic field directly affecting the charge carriers, but rather to an electromagnetic potential in a force-free region.

In order to verify experimentally an Aharonov–Bohm effect for the inertial field, the **B** field must be replaced by an **ω** field. This is achieved by rotating the interferometer in a region with no externally applied fields. In this case, as we now show, the superconducting material becomes a force-free region with a nonzero inertial vector potential, and, thus, corresponds to the rotating ring in the thought experiment.

When a superconductor is rotated, a magnetic field (called the "London moment") is induced in the material which is exactly that of Eq. (5)[13,14] Similarly, an electric field is induced which is exactly that of Eq. (6).[15] Thus, the rotating superconducting material of the Josephson-junction interferometer is a force-free region for the charge carriers (all of which have the same charge to mass ratio) and corresponds to the region $r_1 < r < r_2$ in the thought experiment. Also, since there is no magnetic field induced in the nonconducting region of the SQUID,[16] this part of the interferometer corresponds to the region $r < r_1$ in the thought experiment. The **ω** flux in this region gives rise to an uncanceled inertial vector potential in the force-free region inside the superconducting material. Therefore, the thought experiment of Aharonov and Carmi can be realized experimentally by rotating a Josephson-junction interferometer and measuring the maximum superconducting current. If an inertial vector potential can affect particles in a force-free region, then this current should be periodic as a function of **ω**, with a phase given by Eq. (15).

Remarkably enough, in an experiment performed to measure the Compton wavelength of superconducting electrons, Zimmerman and Mercereau[11] attempted the experiment described above. Unfortunately, they found it difficult to eliminate the magnetic flux through the interferometer due to background fields.[3] In order to circumvent this problem, they rotated the interferometer in an externally applied **B** field. The analysis in this case proceeds as follows: the Hamiltonian is that of Eq. (7) with **A** now composed of a contribution from the external **B** field as well as from the magnetic field induced in the superconducting material. The maximum superconducting current is phase shifted by both the inertial vector potential **a** and the electromagnetic vector potential **A**. The two shifts add linearly, so the maximum superconducting current is given by[11]

$$I = I_0 \left| \cos(\Phi + qB\pi r_1^2/\hbar) \right| \tag{16}$$

with Φ given by Eq. (15). Zimmerman and Mercereau first rotated the apparatus at a *fixed* angular speed ω_1 and then varied the externally applied **B** field to produce an interference pattern described by Eq. (16). Next they rotated the apparatus at a *different fixed* angular speed ω_2 and again varied

[3] For these and some other details about the experiment that do not appear in reference,[11] I am grateful to Prof. Mercereau (private communication).

the applied **B** field to produce a second pattern. The second pattern is shifted from the first by a phase δ, given by

$$\delta = 2m\pi r_1^2(\omega_2 - \omega_1)/\hbar \tag{17}$$

In this way the magnetic flux due to the background fields is eliminated from the measurement and by measuring δ, $(\omega_2 - \omega_1)$, and πr_1^2 Zimmerman and Mercereau were able to find h/m experimentally. On the other hand, by taking h/m as given, we see that the experiment verifies the phase shift Φ as predicted in Eq. (15), and, thus, confirms an Aharonov–Bohm effect in the inertial field.

4. DISCUSSION

In the last section we saw that an experiment performed by Zimmerman and Mercereau[11] confirms an Aharonov–Bohm effect for the inertial field,[10] and, thus, gives the first experimental verification of a physical effect due to a nonelectromagnetic potential in a force-free region.

Because the Hamiltonian for a particle in an inertial field [given in Eq. (3)] is formally equivalent to the Hamiltonian for a charged particle in an electromagnetic field, it is not surprising that there is an Aharonov–Bohm effect in the inertial case. In fact, the formal equivalence of the two Hamiltonians provides the basis for an intuitive approach to making predictions in rotating reference frames, and, in particular, for predicting new effects involving rotating superconductors. One simply takes a known situation that occurs in the electromagnetic case, and constructs an analogous situation in a rotating frame with **A** replaced by **a** and **B** by **ω**. In this way one predicts a Meissner effect in rotating superconductors, quantization of the **ω** flux through a rotating superconducting ring, etc. These and other effects will be elaborated upon in a future paper.

It is interesting to note that this intuitive approach has been successfully applied in the case of the gravitational field. Papini[17] showed that the Hamiltonian for a particle in a weak gravitational field is

$$H = \frac{(\mathbf{P} - mc\mathbf{h}_0)^2}{2m} + \tfrac{1}{2}mc^2 h_{00} \tag{18}$$

where $\mathbf{h}_0$ and h_{00} are gravitational vector and scalar potentials. Because this Hamiltonian is formally equivalent to that for a particle in an electromagnetic field, one expects to find gravitational counterparts of electromagnetic effects, and this is indeed the case.

For example, using this analogy Papini[17] derived an Aharonov–Bohm effect for the gravitational field, and DeWitt[17] showed that gravitational fields in a superconductor are canceled by induced electromagnetic fields, and that the gravitational flux through a superconducting ring is quantized. Thus, the formal equivalence between electromagnetic and gravitational Hamiltonians has already been used to predict interesting gravitational effects by analogy with electromagnetic effects.

In fact, the Hamiltonians for a particle in inertial, electromagnetic, or weak gravitational fields are *all* formally equivalent, so effects occurring in any one case should occur in the others. From this point of view, the experiment described by DeWitt[17] of a mass rotating inside a superconducting ring, is seen to be the gravitational analog of the Zimmerman–Mercereau experiment. Also, as Sakurai[18] pointed out, the neutron interference experiment of Werner *et al.*,[19] which detected a phase shift due to the earth's rotation, can be easily understood by considering the more familiar electromagnetic analog with $\boldsymbol{\omega}$ replaced by $\mathbf{B}$ and $\mathbf{A}$ by $\mathbf{a}$. In this way, new effects in the case of one field can be predicted by looking for analogs to more familiar effects in either of the other two fields.

In conclusion, the Aharonov–Bohm effect for the inertial field is just one example of a formal equivalence between electromagnetic, inertial, and gravitational Hamiltonians which can be used to make a number of interesting predictions by analogy. The experimental verification of an Aharonov–Bohm effect in the inertial field gives additional weight to the predictive power of this approach, underscores the similarity between inertial, electromagnetic, and gravitational potentials, and represents the first experimental verification of an Aharonov–Bohm effect in a nonelectromagnetic field.

ACKNOWLEDGMENTS

I would like to thank Prof. Aharonov and Dr. M. Vardi for many stimulating discussions, and for making many suggestions that improved the manuscript. I would also like to thank Prof. Mercereau for his help with several points concerning the experiment. Finally, it is a pleasure to thank the University of South Carolina for their hospitality and for supporting two visits.

REFERENCES

1. Y. Aharonov and D. Bohm, *Phys. Rev.* **115**, 485 (1959).
2. D. H. Kobe, *Ann. Phys. (N.Y.)* **123**, 381 (1979).

3. Y. Aharonov, H. Pendleton, and A. Peterson, *Int. J. Th. Phys.* **2**, 213 (1969).
4. D. Bohm and B. J. Hiley, *Nuov. Cim.* **52**, 245 (1979).
5. J. H. Harris and M. D. Semon, *Found. Phys.* **10**, 151 (1980).
6. D. Wisnivesky and Y. Aharonov, *Ann. Phys. (N.Y.)* **45**, 479 (1967).
7. P. Tourrenc, *Phys. Rev. D* **16**, 3421 (1977); J. Anandan, *Nuov. Cim.* **53**, 221 (1979).
8. J. S. Dowker, *Nuov. Cim. B* **52**, 129 (1967); G. Papini. *Nuov. Cim. B* **52**, 136 (1967); K. Krauss, *Ann. Phys. (N.Y.)* **50**, 102 (1968); J. Anandan, *Phys. Rev. D.* **15**, 1448 (1977).
9. T. T. Wu and C. N. Yang, *Phys. Rev. D.* **12**, 3845 (1975).
10. Y. Aharonov and G. Carmi, *Found. Phys.* **3**, 493 (1973) (see also the review article of Harris and Semon in Ref. 5).
11. J. E. Zimmerman and J. E. Mercereau, *Phys. Rev. Lett.* **14**, 887 (1965); R. P. Feynman, *Statistical Mechanics* (Benjamin, Reading, Massachusetts, 1972), pp. 303–311.
12. R. C. Jaklevic, J. J. Lambe, A. H. Silver, and J. E. Mercereau, *Phys. Rev. Lett.* **12**, 274 (1964); *Phys. Rev.* **140**, A1628 (1965).
13. R. Becker, F. Sauter, and C. Haller, *Z. Phys.* **85**, 772 (1933); F. London, *Superfluids* (Dover, New York, 1960), Vol. 1, pp. 78–83.
14. M. Bol and W. M. Fairbank, in *Proceedings of the Ninth International Conference on Low Temperature Physics*, part A, J. G. Daunt *et al.* eds. (Plenum Press, New York, 1965); A. F. Hildebrandt, *Phys. Rev. Lett.* **12**, 190 (1964); N. F. Brickman, *Phys. Rev.* **184**, 460 (1969).
15. R. G. Rystephanick, *Am. J. Phys.* **44**, 647 (1976).
16. A. F. Hildebrandt and M. M. Saffren, in *Proceedings of the Ninth International Conference on Low Temperature Physics*, J. G. Daunt *et al.* (Plenum Press, New York, 1965), p. 459.
17. B. S. DeWitt, *Phys. Rev. Lett.* **16**, 1092 (1966); G. Papini, *Phys. Lett.* **24A**, 32 (1967); **23**, 418 (1966).
18. J. J. Sakurai, *Phys. Rev. D.* **21**, 2993 (1980).
19. S. A. Werner, J. L. Staudenmann, and R. Colella, *Phys. Rev. Lett.* **42**, 1103 (1979).