Mathematics 174

Introduction to Auction Project

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What is an Auction?

An auction is...

A market where a fixed pool of buyers (the bidders) compete to purchase... but the price is not set beforehand. The winner is the bidder willing to pay the most!

Different kinds of auctions

- **Sealed-bid** auctions: bidders don't know other bids
  - **First-price**, where the highest bidder pays their bid price
  - **Second-price**, where the highest bidder pays the second-highest bid price
- **English** auctions: bidders bid out loud, until an auctioneer stops it
- **Dutch** auctions: a "bid clock" starts at a high price and ticks down; the first bidder to stop the clock wins

A Model Example: Apple's **iPhone**

Apple's **iPhone** hit the U.S. market in June 2007. Because supplies were so limited on release, iPhones flooded the online auction market immediately.

The question

I want to buy and re-sell an iPhone, to try to make a profit. How should I bid?

We need to know...

- How much can we expect to re-sell for?
- How much, on top of our bid, must we pay to obtain it?
- How much profit is reasonable?

How much of our take can we keep?

By browsing `eBay`, we find that most iPhones sell for around \$800.

A profit margin is...

Our **profit margin** is the percentage of our sales revenue that becomes net profit.
If we re-sell our iPhone for \$800, but have invested \$750 to obtain it, our profit margin is

$$\frac{\$800 - \$750}{\$800} = \frac{\$50}{\$800} = 0.0625 = 6.25\%.$$

Suppose we decide that we want a 10% profit margin on our iPhone. Then our net profit should be

$$\$800 \times (10\%) = \$800 \times (0.1) = \$80.$$

This means we ought to spend a total of

$$\$800 - \$80 = \$720$$

to get it.

Cutting into the profit margin: Costs

So far, we suspect...

Sales revenue	\$800.00	(hopefully)
Extra costs	\$25.00	
Gross profit	\$775.00	
Winning bid	\$720.00	
Net profit	\$80.00	(10%)
Net profit	\$55.00	(6.875%)

The cost of paying your bid is not the only cost you have to worry about! Extra costs — such as shipping and handling — are present and will eat into your profit margin.

Gross vs. net

Net profit refers to the final amount of profit you make — after **all** costs are included.

Before we can decide our bid, we have to take **all** revenue and cost estimates into account.

What is proven value?

In our iPhone example, if we wanted a profit margin of 10%, we would have to have the following balance sheet:

Sales revenue	\$800.00	(hopefully)
– Shipping costs	\$25.00	
Gross profit	\$775.00	
– Winning bid	\$695.00	
Net profit	\$80.00	(10%)

The plan

We figure out the amount we should pay at auction in order to make our fair profit. This amount is called the **projected proven value** of the iPhone.

The problem!

- We don't know if we can actually get \$800 for our iPhone...
- ...so the proven value is unknown at the time of auction!

Playing to win, mathematically

Your team will be bidding against 15 — 20 other companies for a lease giving you the right to drill for oil in a multi-million-acre plot of land in the Gulf of Mexico. The auction will be a first-price, sealed-bid auction conducted by the U.S. Department of Interior.

Uncertainty in our auction process will come from two sources.

Uncertainty 1: The oil's proven value

Before actually drilling, extracting, and selling the oil, we don't know how much our lease will prove to be worth.
However, our company's geologists can *estimate* the revenue and costs for us, and give a projected proven value. This will inform our bidding.

Uncertainty 2: The other bidders' behavior

We want to develop a strategy that will give us a good chance of winning... but the other companies want to win too! What will their strategies be?

Estimating the proven value: Signals

If the (*unknown*) proven value of our oil lease is v million dollars, the estimate our company's geologists make is a **random variable**!

Remember random variables?

A **random variable** X is a quantity whose value is randomly defined.

- If the list of its possible values is finite, X is called a finite random variable.
- If X may take on any value in a continuous range, it is called a continuous random variable.
- Generally, some values of X are more likely to occur than others — this is specified by the probability distribution of X.

The estimated proven value that our geologists give us is called our **signal**, denoted by S_v.

S_v is a continuous random variable and can have any positive value.

Estimating the proven value: Signals

Signals vs. bids

The signal estimates how much you should pay for the tract at auction to make a fair profit. Should we bid this number? **No way!**

- Our geologist might have overestimated the proven value! We might make less than a fair profit — or worse, lose money altogether!
- Plus, it takes the highest bid to win... so if every company bid their signals, the winning company would make money only if **every** geologist underestimated the value. (Not likely.)

Thus, if everyone bids their signals, the winner will ultimately be a loser. We ought to bid less than our signal.

Signal adjustment and bidding strategy

After our iPhone calculations before — estimating that we can re-sell for \$800 and pay \$25 for shipping — we determined a projected proven value (signal) of \$695.00.

Our competing bidders will have different estimates for this — some of which will probably be higher. Besides, we don't want to bid our signal anyway.

Signal adjustment

Our **signal adjustment** is the amount we decide to bid **underneath** our signal.
For instance, if we decide to bid \$650.00 for our iPhone, this is an adjustment of $\$695 - \$650 = \$45.00$.

Bidding strategies

A **bidding strategy** is a decision by a bidder on how much to adjust their signal.

How do we decide on a bidding strategy?

The winner's curse

If every company bid their signal, the winner would be a loser.

The winner's curse

The amount that the winner would lose — that is, the amount by which the winning bidder's signal exceeds the proven value — is called the **winner's curse**.

Example

For instance, if we ultimately bid $695 for our iPhone, and eventually only sold it for $700, then its **actual** proven value was

Sales revenue	$700.00	(actual)
Shipping	$25.00	
Gross profit	$675.00	
Proven value	$605.00	
Net profit	$70.00	(10%)

The winner's curse is then $695.00 - $605.00 = **$90.00**.

If we knew beforehand what the winner's curse would be, we would adjust by that amount. Since we don't *know*, we can estimate.

The Price Is Right strategy: winner's blessing

In another sad twist, the winning bidder always pays more than they need to! This is because they pay too much more than the second-highest bidder. Not only do we want to win the auction, we want to do so by outbidding the competition by as little as possible.

The winner's blessing

The difference between the winning bid and the second-highest bid is called the **winner's blessing**. The winning bidder has always overpaid by this amount.

Example

If we win our iPhone with a bid of \$695, and the second-highest bidder had bid \$680, then we overpaid by

$$\$695 - \$680 = \$15.$$

The winner's blessing in this example is then **\$15.00**.

If we knew beforehand what the winner's blessing would be, we would adjust by that amount. Since we don't *know*, we can estimate.

The uncertainty of the other bidders' behavior

In developing our strategy so far, we can...

- Estimate the proven value of the oil lease to obtain a signal S_v
- Estimate the winner's curse to determine how much we would overpay if our signal won
- Estimate the winner's blessing to determine by how much we can expect a winning bid to win

The game's afoot!

But the other companies all have geologists too, so they have their own signals. What will they do with their signals? What adjustments will they make?

Uniform strategy

To make matters easy, we will assume that all other companies will follow an identical strategy — that is, they will all adjust their signals by the same amount. This is called a **uniform bidding strategy**.

Not all uniform strategies are created equal

Under a uniform bidding strategy, once we figure out what our competitors will do, we can then decide on our best bid.

Two competing forces

In deciding our strategy, we must consider two things:

- If our bid is too low, we will make extra profit if we win... but we probably won't win.
- If our bid is too high, we will be more likely to win... but our profits will be lower.

To balance these forces, we will compute the **expected value** of our strategy — the amount of profit we can make on top of the fair and reasonable amount, weighted by the probability we will win.

The last problem: for a given uniform strategy...

...will the other companies actually be willing to follow it?
They **won't**, if there's any other way that they can raise their expected profit!

Uniform strategy: iPhone example

Suppose I'm bidding against my friend for that blasted iPhone. Say the proven value is \$690, my signal is \$695, and my friend follows a \$20 adjustment strategy.

- If I bid my signal (\$0 adjustment), then I have a 75% chance of winning, but will lose \$5 on the transaction.
  - Expected value: $0.75 \times (-\$5) = -\$3.75$.
- If I make a \$10 adjustment, then I have a 20% chance of winning, and will make \$5 extra profit.
  - Expected value: $0.2 \times (\$5) = \$1$.
- If I make a \$20 adjustment, then I have a 5% chance of winning, and will make \$15 extra profit.
  - Expected value: $0.05 \times (\$15) = \$0.75$.

With enough of these data points, I can figure out what my strategy should be to maximize my expected profit. Suppose the maximum occurs when I adjust by \$10.

The final problem

My friend is doing the same analysis — so she can tell that if she adjusts by \$20, I will adjust by \$10 and probably win! She won't want to adjust by more than I do (nor I her)!

A stable bidding strategy: the Nash equilibrium

When we assume that our competition is doing the same analysis and developing their strategies the same way we are...

The stable bidding strategy

We should find a uniform strategy that both we and our competitors can live with.
In other words... we want a strategy which:

- if our competitors follow it, our expected profit is highest if we follow it too, and
- if we follow it, our competitors' expected profit is highest if they follow it too.

This is called a **stable bidding strategy**.

It is a special case of the Nash equilibrium in economic game theory, the discovery for which John Nash was awarded the Nobel Prize in economics.

Project Assumptions

For the oil lease auction project, your team will be given historical data on 20 previous auctions for oil leases similar to the one you will bid on.

The historical data

For each previous auction, you know the actual proven value, and the signals that each of your competitors received prior to their bid. We do not, however, know what their eventual bid was.

We assume that...

1. The 15 — 20 competing companies you will bid against are your only competitors.
2. All company geologists are equally expert, and on the average, they correctly predict the proven value of oil leases.
3. The distributions of signals for each lease are identical, except for their averages.
    - According to assumption 2, these averages *are* the proven values of the leases.
4. Each company will act in its own best interest, and all companies have equal profit margins and need for business.
    - This ensures that the only reason their proven value estimate differs from yours is due to randomness in the geologists' accuracy.

Your first goal

Goal #1: Analyze the geologists' errors

- The geologists' signals are usually not exactly correct — so figure out how they are distributed.
- Aggregate the signal data across all 20 of your historical leases to figure out how far off a typical signal is from the proven value
- Using this information, simulate 10,000 auctions for your desired lease
- Determine the expected winner's curse and winner's blessing

Your second goal

Goal #2: Simulate some uniform bidding strategies

Figure out...

- If your competitors reduced their signals by the winner's curse amount, what bid gives you the highest expected profit?
  - You will fit a 4th-order polynomial to the data relating your adjustment to your expected profit.
- If your competitors reduced their signals by the winner's curse *and* winner's blessing amounts, what bid gives you the highest expected profit?
- Repeat these steps for many more uniform bidding strategies of your choosing.

Goal #3: Find the Nash equilibrium

From the data you found in goal #2, determine the bidding strategy which meets both of these conditions:

- You and your competitors make the same adjustment, and
- When your competitors make this adjustment, your profit is maximized when you do too.

You will again fit a polynomial to the data relating your competitors' adjustment to your optimal adjustment.

Go for it!