Roller Coasters: Theory, Design, and Properties
Short Term 2005
Rolling Ball Coasters: In-Class Problems

Cross Product

1. Compute $\langle 0, -2, -3\rangle \times \langle 4, 0, 2\rangle$.

2. Compute $\langle 4, 0, 2\rangle \times \langle 0, -2, -3\rangle$.

3. Notice the relationship between your answers in numbers (1) and (2). Does this relationship occur in general? To answer this, find expressions for $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$. How are these two expressions related?

4. In this problem we will look for a geometrical picture for the cross product.
   a) Find $\langle 8, 0, 0\rangle \times \langle 3, 2, 0\rangle$.

   b) Compute $\Big|\langle 8, 0, 0\rangle \times \langle 3, 2, 0\rangle\Big|$.

   c) Draw these vectors. If we look at them as two sides of a parallelogram, draw in the other edges.

   d) Find the area of the parallelogram in part *c*.

Torque

5. What is the magnitude of the torque caused by a 4 N force applied perpendicularly at a distance 3 meters from the axis of rotation?

6. A 12 N force is applied at an angle $\pi/6$, 2 meters from the axis of rotation. At what distance d does a 2 Newton (perpendicular) force cause a torque with the same magnitude?

7. A sphere with radius R is rolling down a track, such that the center of mass is b meters above the rails. If the translation kinetic energy is 10 Joules, find the rotational kinetic energy. (Assume no slipping.) Your answer should contain only R and b as undetermined constants.

8. In the lecture, we found that the acceleration of a sphere on a two-rail track is given by the equation

$$a_C M = \frac{g \sin\theta}{\frac{2R^2}{5b^2} + 1}.$$

   a) What is the acceleration as $b \longrightarrow R$?

   b) What is the acceleration as $b \longrightarrow 0$?