Roller Coasters: Theory, Design, and Properties
Short Term 2005
Rolling Ball Coasters Homework

1. Find $\langle 1, 0, 0\rangle \times \langle 0, 1, 0\rangle$.

2. Find $\langle 2, 1, -3\rangle \times \langle 1, -2, 2\rangle$.

3. In this problem you will derive a more general version of the last formula we did in class.
   a) Let $f(x)$ be a function. Parameterize this curve.
   b) Assume that $f(0) = 0$ and that the particle starts at rest. If $h = 0$ at the origin, find the initial total mechanical energy.
   c) Find the final potential energy as a function of the parameter introduced in part (*a*).
   d) Now assume that the particle has a moment of inertia I, and that it rolls in such a way that $v = \omega b$. What is the total kinetic energy of the particle as a function of v^2? (This equation should have only m, I, and b as undetermined constants, and $f(s)$ should also appear.)
   e) Setting the answer from part (*b*) equal to the sum of parts (*c*) and (*d*), solve for v^2. This term will be useful when finding the centripetal force.

   Now we look at a special case. Let $f(s) = s\sin\theta$ for some fixed θ. This will give a line that is inclined at an angle θ.

   f) Find v^2 as a function of s.
   g) Find κ as a function of s.
   h) What is the centripetal force on this track? Does this make sense?
   g) Consider the following relation, which holds when the acceleration is constant and s is the real change in distance along the track (which is the case in this particular example):
   $$a = \frac{v^2}{2s}.$$
   Using this formula and your answer from part (*f*), find the acceleration.
   i) In the case of a rolling sphere, we know the moment of inertia I. Plugging this in, compare your answer to the result from class.