Roller Coasters: Theory, Design, and Properties
Short Term 2005
Intermediate Math: In-Class Problems

Parametric Equations

1. Consider the following parameterization of the unit circle.

   $$\langle \cos s, \sin s \rangle$$

   Find the unit tangent vector as a function of s.

2. Find the parameterization for an ellipse with one semi-axis of length 2 and the other semi-axis of length 3.

3. Take the previous parameterization for an ellipse, and turn it into a elliptical spiral around the z-axis.

4. Find a parameterization for the negative sine curve.

5. Find a different parameterization for the negative sine curve.

6. Consider some function of the form $y = f(x)$. Write this as a parameterization.

7. Consider the following curve:

$$\langle \sin s + \frac{s}{3}, 1 - \cos s \rangle$$

a) Plot this curve using Mathematica, letting s vary from 0 to π. Call this curve *loop*.

b) Find a tangent vector as a function of s.

c) Turn the vector you found in part (*b*) into a unit vector. Define it as $\hat{d}(s)$.

d) Find the force of gravity in the tangent direction, and use it to define a function $grav(s)$. (This should give a scalar function of s)

e) Set the value of g to 9.81 and the mass m to .05 kg. Plot $grav(s)$ from $s = 0$ to $s = \pi$. Does the shape of this match your intuition? Where does it change sign? Does that makes sense?

f) Plot a vector $grav(s)\hat{d}$ where the tail is at the position $\langle \sin s + \frac{s}{3}, 1 - \cos s \rangle$ for $s = \{0, \frac{\pi}{8}, \frac{2\pi}{8}, \frac{3\pi}{8}, \frac{4\pi}{8}, ..., \frac{15\pi}{8}, 2\pi\}$. Display this set of vectors on top of the graphic "loop".