Roller Coasters: Theory, Design, and Properties
Short Term 2005
Basic Math: In-Class Problems

Vectors

Magnitude

1. Find the magnitude of $\vec{r}$ where $\vec{r}$ is $\langle 1, 3\rangle$.

2. Let $\mathbf{r} = \langle 3, 4\rangle$. Find the magnitude of $\mathbf{r}$.

3. Find the magnitude of $\langle 1, 1, 3, 1\rangle$.

4. Evaluate $|\langle 6, -3, 2\rangle|$.

Addition/Subtraction

5. If $\vec{u} = \langle 1, 0\rangle$ and $\vec{v} = \langle 0, 1\rangle$, then find $\vec{u} + \vec{v}$.

6. Find the sum of $\langle 10, -5, 3\rangle$ and $\langle 0, -9, 7\rangle$.

7. If $\vec{u} = \langle 1, 0, 0 \rangle$ and $\vec{v} = \langle 0, 0, 1 \rangle$, then find $\vec{u} - \vec{v}$.

8. Evaluate $|\langle 7, -3, 0 \rangle - \langle -9, 5, 2 \rangle|$.

Multiplication by Scalars

Multiply the given vector by the given scalar.

9. $\vec{r} = \langle 1, 3 \rangle$, $a = \frac{1}{3}$

10. $\vec{r} = \langle 2, 3 \rangle$, $a = \frac{1}{4}$

11. $\vec{r} = \langle 1, 2, 2.5 \rangle$, $a = 2$

Unit Vectors

12. Find the unit vector in the direction of $\langle 1, 1 \rangle$.

13. What is the magnitude of the unit vector in the direction $\langle 12, 14, \pi \rangle$?

14. What is the y-component of the unit vector in the direction $\langle \sqrt{3}, 1 \rangle$?

15. What is the unit vector in the direction of $\langle 5, 3, 3, \sqrt{6} \rangle$?

Dot Product

16. Evaluate $\langle 2, 4, 1\rangle \cdot \langle 1, -1, 4\rangle$.

17. Find the angle between the vectors $\langle -2, \sqrt{3}\rangle$ and $\langle -3\sqrt{3}, 1\rangle$.

18. Find the angle between the vectors $\langle 2, 1, 1\rangle$ and $\langle 1, 2, -1\rangle$.

19. Find $\langle 2, 3, 4\rangle \cdot \langle 0, 1, 0\rangle$. What is the effect of this operation in this example?

20. Using the last example as a model, determine the magnitude of the vector $\langle 1, 2\rangle$ in the direction $\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\rangle$.

21. The direction vector in the last example had magnitude 1. What would happen if it was not a unit vector?

Put it all together

We now have a technique for finding the component of some vector in a given direction. This can be used to find the component of gravity along and perpendicular to a rollercoaster track. This is precisely what we will do now.

The following equations give the tangent and normal vectors to a concave down parabola. (For a given value of s, the tangent vector is tangent to the parabola; the normal vector is perpendicular to the parabola.)

$$\begin{aligned}\text{Tangent Vector} &= \langle 1, -2s \rangle \\ \text{Normal Vector} &= \langle 2s, 1 \rangle\end{aligned}$$

The value of s will change as we move along the path. For now, just assume it is an arbitrary constant.

a) Using the given equations, find the unit vector in the tangent and normal directions.
b) Check to make sure the normal and tangent directions are perpendicular.
c) Using your answer from part (*a*), find the component of gravity, $\vec{g}$, in the tangent and normal directions. Use $\vec{g} = \langle 0, -mg \rangle$.
d) Use your results from parts (*a*) and (*c*) to determine the component vectors of gravity in the tangent and normal directions. Check to make sure that the magnitude of their vector sum is still mg.
e) What are the two component forces when $s = \frac{1}{2}$? What is the significance of this point?