Roller Coasters: Theory, Design, and Properties
Short Term 2005
Basic Math Homework: Vectors

1. Find the magnitude of $\mathbf{r}$ if $\mathbf{r} = \langle 0, 0, 5, 0 \rangle$.

2. Given $\vec{r} = \langle 5, 4, 3 \rangle$, find the magnitude of $\vec{r}$.

3. Evaluate $|\langle -8, 1, 4 \rangle|$.

4. Let $\vec{u} = \langle 6, 11, 3 \rangle$ and $\vec{v} = \langle 0, 5, 2 \rangle$, then find $\vec{u} + \vec{v}$.

5. Let $\vec{u} = \langle 9, 5, 7, 2 \rangle$ and $\vec{v} = \langle 8, -4, 1, -6 \rangle$, then find $\vec{u} - \vec{v}$.

6. Let $\vec{u} = \langle 3, -2 \rangle$ and $\vec{v} = \langle 3, 10 \rangle$, then find $|\vec{u} + \vec{v}|$.

7. Let $\vec{u} = \langle 10, -5, 1 \rangle$ and $\vec{v} = \langle -2, 11, 0 \rangle$
   a) Find $|\vec{u} + \vec{v}|$.

   b) Find $|\vec{u}| + |\vec{v}|$.

8. Let $\vec{r} = \langle 15, 12, 4 \rangle$, $a = \frac{1}{2}$. Find $a\vec{r}$.

9. Let $\vec{r} = \langle 2, 1 \rangle$, $a = \frac{1}{\sqrt{5}}$. Find $a\vec{r}$.

10. Find the unit vector in the direction
    a) $\langle 2, 1 \rangle$.

    b) $\langle 3, 4 \rangle$.

    c) $\langle 5, 12 \rangle$.

d) $\langle 2, 1, 2\rangle$.

e) $\langle 3, 2, 2\rangle$.

11. Evaluate $\langle 1, 2, 1\rangle \cdot \langle 3, -1, 4\rangle$

12. Evaluate $\langle 4, 1\rangle \cdot \langle -\frac{1}{2}, 2\rangle$. Describe the orientation of these two vectors.

13. Find the angle between the vectors $\langle -2, \sqrt{3}\rangle$ and $\langle -5, -\sqrt{3}\rangle$

14. The following equations give the tangent and normal vectors to a (negative) sine curve.

$$\begin{aligned} \text{Tangent Vector} &= \langle 1, -\cos s\rangle \\ \text{Normal Vector} &= \langle \cos s, 1\rangle \end{aligned}$$

a) Using the given equations, find the unit vector in the tangent and normal directions.
b) Check to make sure the normal and tangent directions are perpendicular.
c) Using your answer from part (*a*), find the component of gravity, $\vec{g}$, in the tangent and normal directions. Use $\vec{g} = \langle 0, -mg\rangle$.
d) Check to make sure that the magnitude of this vector is still mg.
e) Sketch the curve $y = -\sin s$. (In this sketch, the s-axis replaces what we would usually call the x-axis) Find the magnitude of gravity in the tangent and normal directions at the points $s = 0$, $s = \frac{\pi}{2}$, and $s = \pi$. Check these with your sketch.