Mathematics s21 2007 - Day Three

Today we finally begin to write proofs. Writing a proof is not the same as solving an equation. A proof is a sequence of logical steps, usually written in complete sentences using proper notation, often with logical connections between the steps to help the reader. We will learn how to write, read, and critique such proofs.

1. 9:00-10:20
   (a) Students present problems 1, 2(abcd) from homework.
   (b) In groups of three, read sections 1.9, 1.10 (find typo on page 30), 1.11 (find typo on page 31), and 1.12, doing all exercises in the text.
2. 10:20-10:35 Break
3. 10:35-11:30
   (a) In same groups, write direct proofs of the following.
      i. The product of two even integers is even.
      ii. There is a positive real number that is greater than its square.
      iii. The product of two odd integers is odd.
      (Definition: m is odd if $m = 2k + 1$ for some integer k.)
      iv. The sum of two odd integers is even.
   (b) Groups present selected exercises.
4. 11:30-11:55 LaTeX
5. 11:55-1:00 Lunch
6. 1:00-3:00
   (a) In new groups of three, read through sections 1.13, 1.14 and 1.15, doing all exercises in the text.
   (b) Use proof by contrapositive to prove the following (which are much harder to do by direct proof - try it yourself!) [The numbers x, a, b, p, and q are integers.]
      i. If x^2 is odd, then x is odd.
      ii. If ab is even, then a is even or b is even.
      iii. If pq is odd, then p is odd and q is odd.
   (c) Use proof by contradiction to prove the following.
      i. $\frac{n}{n+1} > \frac{n}{n+2}$ for all natural numbers n (that is, integers $n > 1$).
      ii. If x is irrational, then $2007x$ is irrational. [The number x is real.]
   (d) Groups present selected exercises.

Today's Key Ideas:
proof of existence
proof of uniqueness
counterexamples
direct proof
proof by contrapositive (Proving $A \Rightarrow B$ is equivalent to proving $(\sim B) \Rightarrow (\sim A)$.)
proof by contradiction (To prove P, begin by assuming $\sim P$. Then arrive at some contradiction: $Q \wedge \sim Q$.)

Homework

1. Prove that there exist integers p and q such that $2p + 7q = 1$.
2. Prove that the cube of an odd integer must be odd.
3. Prove by contrapositive that if x^2 is not divisible by 4, then x is odd.
4. Prove by contradiction that there are infinitely many prime numbers.
5. Read sections 2.1 and 2.2.