

Name: _____

Mathematics 110 - Fall 2005
Lab Five - Chaos

A common way to predict the growth or decline of a population is the **logistic model**, which uses the population in year t to predict the population in the next year, year $t + 1$. The model employs the following equation to do this.

$$P_{t+1} = \lambda P_t(1 - P_t)$$

In this formula, P_t is the population in year t represented as a *fraction* of a theoretical maximum population. For the sake of concreteness, let's suppose we are studying a lake that has a maximum capacity of 1,000,000 fish; thus, $P_3 = .752243$ means that in year 3, there were 752,243 fish in the lake (75.2243% of 1,000,000).

The symbol λ (lambda) is a constant that is determined by the environment and the species being studied.

From the formula above, we can see that this model is **deterministic** because the population in any year (called P_{t+1}) is completely determined by the population in the previous year (P_t), which was itself determined by the population in the previous year and so on back to whatever the initial population (P_0) was.

Exercise 1: Applying the Model. Find the annual fish population for each of the first 300 years for each choice of λ and the two nearly identical initial populations in the table below. (Recall that $P_0 = .600000$ means that the study starts with 600,000 fish in the lake, while $P_0 = .600001$ corresponds to an initial population of 600,001 fish, a difference of just a single fish among 600,000.)

The easiest way to complete such a table is by using a spreadsheet, such as Microsoft Excel.

- Begin by entering the word "Year" in cell A1 (upper left corner). In the cell below that (A2), enter the number 0. Now, rather than entering by hand the numbers 1 through 300, we can use a formula to have Excel do the work for us. In cell A3, enter the following: =A2+1. Then copy this cell and paste it into the 299 cells below. This formula uses a "relative" reference, so when you paste it, each new cell will simply use the value of the cell above it rather than the value of A2.

Pick a cell (say A17) and look at its formula to be sure you understand how relative references work.

- Next, enter the values for λ (just the numbers 1.5, 1.5, and so on - no λ symbols) in the cells B1 through G1 and the initial populations in the row below that. To display 6 decimal places, go to the pull-down menu labelled "Format", then choose "Cells" and then the category "Number".
- Now, in cell B3 enter the formula for the logistic equation: =B\$1*B2*(1-B2). The dollar sign before the 1 makes it an "absolute" reference, so that when you copy and paste this formula into the 299 rows below, each new cell will always look back to row 1 for the value of λ but will only look one cell above to find the value for the previous year's population (a relative reference).
- Finally, you can copy cells B3 through B302 (as a single block) into the five other columns.

Year	$\lambda = 1.5$	$\lambda = 1.5$	$\lambda = 3.5$	$\lambda = 3.5$	$\lambda = 4.0$	$\lambda = 4.0$
0	.600000	.600001	.600000	.600001	.600000	.600001
1						
2						
⋮						
299						
300						

Exercise 2: Examining the Data for Long Term Behavior. As we look ahead in time, several things may happen to the population.

- It may settle down and approach (or even reach) one particular value.
- It may cycle back and forth among two or more values.
- It may behave erratically, displaying no apparent pattern at all.

For each situation in Exercise 1, decide which of the above three possibilities occurs. In cases where the population cycles among several values, identify the **period** of this oscillation (that is, the number of values among which the population cycles).

- (a) $\lambda = 1.5, P_0 = .600000$
- (b) $\lambda = 1.5, P_0 = .600001$
- (c) $\lambda = 3.5, P_0 = .600000$
- (d) $\lambda = 3.5, P_0 = .600001$
- (e) $\lambda = 4.0, P_0 = .600000$
- (f) $\lambda = 4.0, P_0 = .600001$

Exercise 3: The Effect of the Initial Condition. In each case, indicate if the small change made to the initial population (the one extra fish) had a significant effect on future values of the population.

- (a) $\lambda = 1.5$
- (b) $\lambda = 3.5$
- (c) $\lambda = 4.0$

Exercise 4: Experimenting with the Data.

- (a) For each value of λ , try several (at least five) different initial populations (numbers greater than 0 and less than 1). By doing so, can you significantly alter the long-term behavior of the population? Give an answer for each of the three values of λ .
- (b) Find a value of λ that will result in a population that cycles with each of the following periods.
 - (i) period 2
 - (i) period 8
 - (i) period 16 (you may need to add about 300 more rows in order to see this one)
- (c) What is the smallest λ you can find for which the population cycles rather than settling down to one value?